GRADE 12 EXAMINATION
NOVEMBER 2010

ADVANCED PROGRAMME MATHEMATICS

MARKING GUIDELINES

Time: 3 hours 300 marks

These marking guidelines were used as the basis for the official IEB marking session. They were prepared for use by examiners and sub-examiners, all of whom were required to attend a rigorous standardisation meeting to ensure that the guidelines were consistently and fairly interpreted and applied in the marking of candidates' scripts.

At standardisation meetings, decisions are taken regarding the allocation of marks in the interests of fairness to all candidates in the context of an entirely summative assessment.

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MODULE 1 CALCULUS AND ALGEBRA

QUESTION 1

(i) Prove true for n = 1: LHS = 1 RHS = 1
$\therefore$ Statement true for n = 1

(ii) Assume statement true for n = k
i.e. $1+2\left(\frac{1}{2}\right)+3\left(\frac{1}{2}\right)^2 + ... + k\left(\frac{1}{2}\right)^{k-1} = 4-\frac{k+2}{2^{k-1}}$

(iii) Prove true for n = k + 1:

LHS $= 1+2\left(\frac{1}{2}\right)+3\left(\frac{1}{2}\right)^2 + ... + k\left(\frac{1}{2}\right)^{(k-1)} + (k+1)\left(\frac{1}{2}\right)^k$

RHS $= 4-\frac{k+1+2}{2^{k+1-1}}$

$= 4-\frac{k+3}{2^k}$

$= 4-\frac{k+2}{2^{(k-1)}} + (k+1)\left(\frac{1}{2}\right)^k$

$= 4-\frac{2k+4-k-1}{2^k}$

$= 4-\frac{k+3}{2^k}$

= RHS

(iv) $\therefore$ Statement true for n = 1
If the statement is true for n = k, it is also true for n = k + 1
$\therefore$ Statement true for all n $\in$ IN.

[15]

QUESTION 2

2.1 $T = T_s + (T_0 - T_s)\, e^{-kt}$

$60 = 20 + (90 - 20)\, e^{-10k}$
$40 = (70)\, e^{-10k}$
$\frac{4}{7} = e^{-10k}$
$\therefore -10k = \ell n \frac{4}{7}$
$\therefore k = -\frac{1}{10}\ell n \frac{4}{7}$ (8)

2.2 $T = 20 + 70\, e^{-k\,.\,15}$
$= 50^o$ C (3)

2.3 T = 20 (2)

2.4 This is the temperature that the soup can reach after a very long time. (2)

[15]

QUESTION 3

3.1 $|x|^2 - 4|x| - 5 = 0$

$x \geq 0$ OR $x < 0$

$x^2 - 4x - 5 = 0$ | $x^2 + 4x - 5 = 0$

$(x - 5)(x + 1) = 0$ | $(x + 5)(x - 1) = 0$

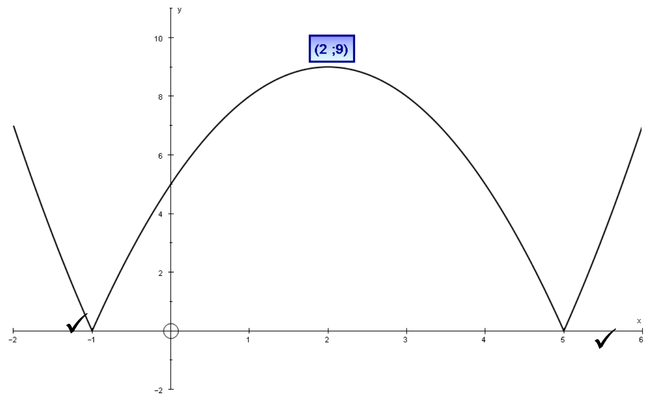
$x = 5$ or $x = -1$ (NA) | $x = -5$ or $x = 1$ (NA) (8)

Alternate: $(|x| - 5)(|x| + 1) = 0$

$|x| = 5$ of $|x| = -1$

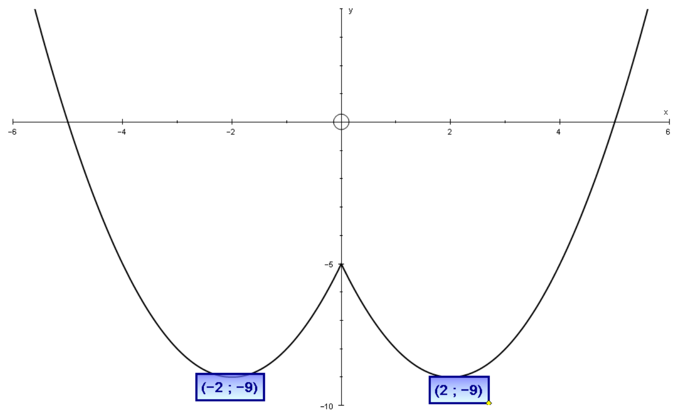
$\therefore x = \pm 5$ n.a.

(a)



(4)

(b)



(7)

[19]

QUESTION 4

4.1 1 – i also a zero

Sum = 2 and Product = $1 - i^2 = 2$

$\therefore x^2 - 2x + 2$ a factor

$\therefore f(x) = (x^2 - 2x + 2)(x - 3)$

$= x^3 - 5x^2 + 8x - 6$

$\therefore a = -5$; $b = 8$

OR $(x - 1 + i)(x - 1 - i)$

$= x^2 - 2x + 1 - i^2$

$\therefore x^2 - 2x + 2$ a factor

OR:

$x = 1 + i$

$x - 1 = i$

$x^2 - 2x + 1 = -1$ (7)

$\therefore x^2 - 2x + 2$ is a factor

4.2 $x = 3$ (2)

4.3 $x > 3$ (2)

[11]

QUESTION 5

5.1 (a) $\lim\limits_{x \to 3} g(x) = \lim\limits_{x \to 3}\left(x^2 + 1\right)$

$= 10 \quad \therefore$ continuous

$= g(3)$ (4)

5.1 (b) $\lim\limits_{x \to -1^+} g(x) = 2$ and $\lim\limits_{x \to -1^-} g(x) = 0$

$\therefore \lim\limits_{x \to 1} g(x)$ does not exist

$\therefore$ Jump discontinuity (5)

5.2

x		0		1		3	
$f'(x)$	–		–		–		+

At x = 0, point of inflection as $f'(x) < 0$ for x < 0 and x > 0

At x = 3, Minimum Turning point as change in sign from neg to pos (7)

[16]

QUESTION 6

6.1 $f(x+h) = \dfrac{1}{1 - 2(x+h)}$

$$f(x+h) - f(x) = \frac{1}{1-2(x+h)} - \frac{1}{1-2x}$$

$$= \frac{1 - 2x - 1 + 2x + 2h}{\left(1 - 2(x+h)\right)(1-2x)}$$

$$= \frac{2h}{\left(1 - 2(x+h)\right)(1-2x)}$$

$$f'(x) = \lim_{h \to 0} \frac{2h}{\left(1 - 2(x+h)\right)(1-2x)} \cdot \frac{1}{h}$$

$$= \frac{2}{(1-2x)^2}$$ (8)

6.2 $\dfrac{dy}{dx} = \cos y . \dfrac{dy}{dx} . \sin x + \sin y . \cos x$

$$\frac{dy}{dx}\left(1 - \cos y . \sin x\right) = \sin y \cos x$$

$$\frac{dy}{dx} = \frac{\sin y . \cos x}{1 - \cos y . \sin x}$$ (8)

6.3 (a) $y' = \sec^2 x$

$y'' = 2\sec x \quad (\sec x \tan x) \qquad = 2\sec^2 x \tan x$

(b) LHS = $y'' - 2y$

= $2\sec^2 x\tan x - 2\tan x$

= $2\tan x(\sec^2 x - 1)$

= $2\tan x(\tan^2 x)$

= $2\tan^3 x$

= $2y^3$ = RHS (10)

[26]

QUESTION 7

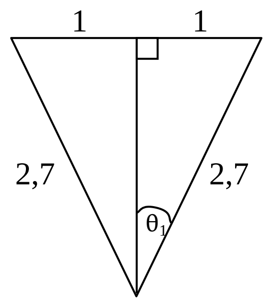
7.1 In ΔBPE: $\cos\theta = \dfrac{BP^2 + EP^2 - BE^2}{2(BP)(EP)}$

$\therefore \cos\theta = \dfrac{2,7^2 + 2,7^2 - 2^2}{2(2,7)(2,7)}$

$\therefore \cos\theta = 0,725\ \ldots$

$\therefore \theta = 0,759$

OR



$\sin\theta_1 = \dfrac{1}{2,7} \qquad \theta_1 = 0,3794$

$\therefore \theta = 0,759$ (5)

7.2 Area BECD = 2 x 3 = 6m^2

Area sector BPE $= \dfrac{1}{2} r^2\ \theta$

$= \dfrac{1}{2}(2,7)^2(0,76)$

$= 2,7702$ m^2

Area $\Delta BPE = \dfrac{1}{2} r^2 \sin\theta$

$= \dfrac{1}{2}(2,7)^2 \sin(0,76)$

= 2,511 … m^2

$\therefore$ Area of $\overset{\frown}{BQE}$ = 0,2590 …

$\therefore$ Area window = 6 + 0,2590 m^2 = 6,26 m^2 (9)

[14]

QUESTION 8

8.1

(a) $\int (2-3x)^{-\frac{1}{2}} \quad dx = \dfrac{(2-3x)^{\frac{1}{2}}}{\frac{1}{2} \quad (-3)} \ + \ c$ **penalty of 1 to apply only once if no '*c*'** (5)

(b) $\int x \sin x \, dx = x \quad (-\cos x) \quad - \int (-\cos x) \qquad dx$

$= -x\cos x + \sin x \quad + c$ (5)

(c) $\int x \cos(x^2)\, dx = \dfrac{\sin \quad (x^2)}{2} \ + \ c$ (5)

OR: let u = x^2

$\therefore$ du = 2xdx

$\therefore$ integral = $\dfrac{1}{2} \quad \sin \quad (x^2) \quad + \ c$

8.2 $\Delta x_i = \dfrac{2}{n}$

$x_i = \dfrac{2i}{n}$

$f(x_i) = \left(\dfrac{2i}{n}\right)^2 \ + \ \left(\dfrac{2i}{n}\right)$

$= \left(\dfrac{4i^2}{n^2}\right) + \dfrac{2i}{n}$

$f(x_i)\Delta x_i = \dfrac{8i^2}{n^3} + \dfrac{4i}{n^2}$

$\sum_{i-1}^{n} f(x_i)\,\Delta x_i = \sum_{i-1}^{n}\left(\dfrac{8i^2}{n^3} + \dfrac{4i}{n^2}\right)$

$= \dfrac{8}{n^3} \cdot \left(\dfrac{n}{6}(2n+1)(n+1) \quad + \dfrac{4}{n^2} \cdot (n+1)\right)$

$= \dfrac{8}{3} \checkmark + \dfrac{4}{n} + \dfrac{4}{3n^2} + 2 \quad + \dfrac{2}{n}$

$\int_0^2 (x^2 + x)\,dx = \lim_{n\to\infty} \sum f(x_i)\Delta x_i$

$= \lim_{n\to\infty}\left[\dfrac{8}{3} + \dfrac{4}{n} + \dfrac{4}{3n^2} + 2 + \dfrac{2}{n}\right]$

$= \dfrac{8}{3} + 2 = \dfrac{14}{3} = 4,67$ (13)

[28]

QUESTION 9

9.1 y = −1,44 (2)

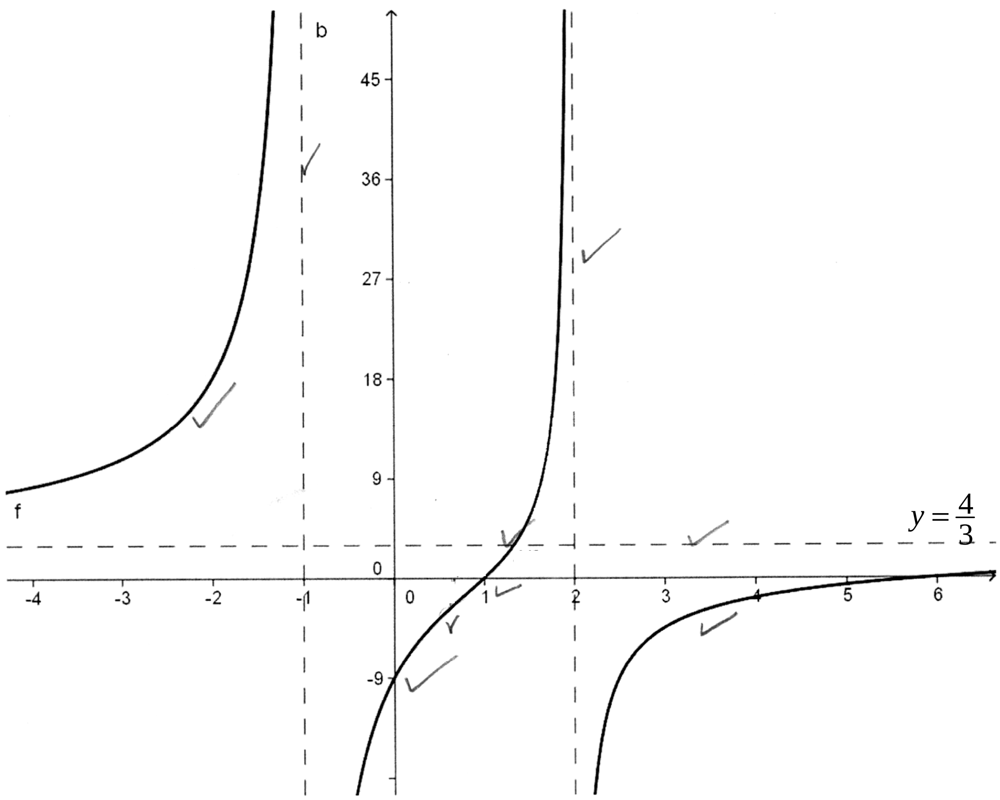
9.2 X – ints : (6 ; 0) and (1 ; 0)
Y – int: (x = 0) $\therefore y = -9$ ie (0; –9)
Asymptotes: Vertical : x = 2 ; x = – 1
Horizontal; y = 3
(No oblique asymptotes) (7)

9.3 $\frac{3(x-6)(x-1)}{(x-2)(x+1)} = 3$

$\therefore x^2 - 7x + 6 \quad = x^2 - x - 2$

$\therefore x = \frac{4}{3}$ (5)

9.4



(9)
[23]

QUESTION 10

10.1

(a)

$$TS^2 = x^2 - 4$$

$$\therefore TS = \sqrt{x^2 - 4}$$

$$\therefore SH = 6 - \sqrt{x^2 - 4} \qquad (2)$$

(b)

$$t = \frac{\text{distance}}{\text{speed}} = \frac{x}{3} + \frac{6 - \sqrt{x^2 - 4}}{5}$$

For shortest time:

$$t' = 0$$

$$\therefore \frac{1}{3} - \frac{1}{2} \cdot \frac{\left(x^2 - 4\right)^{-\frac{1}{2}}}{5} \cdot 2x = 0$$

$$\therefore 5 - \frac{3x}{\sqrt{x^2 - 4}} = 0$$

$$\therefore 25 = \frac{9x^2}{x^2 - 4}$$

$$\therefore 16x^2 = 100$$

$$\therefore 4x = 10$$

$$\therefore x = 2{,}5 \text{ km} \qquad (10)$$

10.2

(a)

$$y = \tan\frac{\pi}{3} \sec\frac{\pi}{3}$$

$$= 2\sqrt{3}$$

$$\therefore A\left(\frac{\pi}{3}; 2\sqrt{3}\right) \qquad B\left(\frac{2\pi}{3}; 2\sqrt{3}\right) \qquad (4)$$

(b)

$$A = 2 \int_0^{\frac{\pi}{3}} f(x)\,dx + \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} 2\sqrt{3}\,dx \qquad (4)$$

OR

$$A = \int_0^{\frac{\pi}{3}} f(x)\,dx + \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} 2\sqrt{3}\,dx + \int_{\frac{2\pi}{3}}^{\pi} f(x)\,dx$$

OR

$$A = 2 \int_0^{\frac{\pi}{3}} f(x)\,dx + \left(\frac{\pi}{3}\right) \square 2\sqrt{3}$$

(c) $2\sec x\Big|_0^{\pi/3} + 2\sqrt{3}x\Big|_{\frac{\pi}{3}}^{2\pi/3}$

$= (2-1) \quad + \quad \dfrac{2\sqrt{3}\pi}{3}$

$= 2+\dfrac{2\sqrt{3}\pi}{3}$ (6)

10.3 $\pi\int_0^a (\tan x \sec x)^2\, dx = \dfrac{\pi}{3}\tan^3 x\Big|_0^a = \dfrac{\pi}{3}$

$\therefore \tan^3 a = 1$

$\therefore a = \pi/4$ (7)

[33]

Total for Module 1: 200 marks

MODULE 2 STATISTICS

QUESTION 1

X~N(27;2,5^2)

1.1 90 %: $27 \pm 1,645 \times 2,5$ NB: Whole population

(22,8875;31,1125) (3)

1.2 Sample: $\overline{x} = 26,15$ $\sigma_{n-1} = 4,3923$ $n = 20$

$\therefore \mu = \overline{x} \pm 1,96 \frac{\sigma_{n-1}}{\sqrt{n}}$

$\therefore \mu \in (24,225;28,075)$ (8)

1.3 $H_o : \mu = 27$

$H_1 : \mu < 27$ NB : 1-sided

$$\therefore z = \frac{\overline{x} - \mu}{\frac{\sigma_{n-1}}{\sqrt{n}}} = \frac{26,15 - 27}{\frac{4,3923}{\sqrt{20}}} = -0,8654$$

Critical value : -1,645

$\therefore$ Not enough evidence to reject the null hypothesis thus accept null hypothesis. (10)

[21]

QUESTION 2

2.1 Number of ways of arranging letters is $\frac{11!}{2!2!} = 9979200$ as the I's and B's are repeated.

Number of ways with the I's together is $\frac{10!}{2!} = 1\,814\,400$

Thus number of ways with I's separate is 9 979 200 – 1 814 400 = 8 164 800

Thus probability of I's separate is $\frac{8\,164\,800}{9\,979\,200} = \frac{9}{11}$ = $0,8182$ (6)

2.2 Number of ways of choosing 2 vertices from $^{15}C_2$ = 105 but this has included the edges of the polygon thus the number of diagonals is 105 – 15 = 90.

OR 15 $\times$ 12 diagonals $\div$ 2 (for duplicates) = 90 (4)

[10]

QUESTION 3

3.1 $\bar{x} = \dfrac{\sum x_i}{n}$

$\therefore \dfrac{31}{3} = \dfrac{124}{n}$ thus $n = \dfrac{124}{\frac{31}{3}} = 12$ (2)

3.2 $\bar{y} = \dfrac{\sum y_i}{n} = \dfrac{124}{12} = \dfrac{31}{3}$ or $10\frac{1}{3}$ (2)

3.3 $b = \dfrac{12 \times 1045 - 124 \times 124}{12 \times 1578 - (124)^2} = -0,7966$

$\therefore y = a - 0,7966x$ and passing through the point $\left(\frac{31}{3}; \frac{31}{3}\right)$

$\frac{31}{3} = a - 0,7966 \times \frac{31}{3}$

$\therefore$ a = 18,5649

$\therefore$ y = 18,5649 – 0,7966x (8)

3.4 y = 18,5649 – 0,7966 × 9
= 11,3955 (2)

3.5 $r = 0,7966 \times \dfrac{\sqrt{24,72}}{\sqrt{23,88}} = -0,8104$ (4)

3.6 A relatively moderate negative linear relationship. (2)

[20]

QUESTION 4

4.1 (a) $\mathrm{P}(x \le 1) = \mathrm{P}(x = 0) + \mathrm{P}(x = 1)$

$= a\left(\frac{2}{3}\right)^0 + a\left(\frac{2}{3}\right)^1$

$= a + \frac{2}{3}a = \frac{5}{3}a$ (3)

(b) $\sum_{x=0}^{\infty} \mathrm{P}(\mathrm{X} = x) = \dfrac{a}{1-r} = \dfrac{a}{1-\frac{2}{3}} = 1$ (5)

$\therefore a = \frac{1}{3}$

4.2 P(boy) = 0,5 = P(girl)
Let X = the number of girls then X ~ Bin (7;05)
$\mathrm{P}(x = 0) = {}^7C_0 \times 0,5^0 \times 0,5^7$
$\mathrm{P}(x = 1) = {}^7C_1 \times 0,5^1 \times 0,5^6$
$\mathrm{P}(x = 2) = {}^7C_2 \times 0,5^2 \times 0,5^5$
Thus $\mathrm{P}(x \le 2) = 0,5^7 \times (1 + 7 + 21) = 0,2266$
So $\mathrm{P}(x \ge 3) = 1 - 0,2266 = 0,7734$ (10)

4.3 Hypergeometric: where x is the number of computers with a virus.

$\therefore \text{P}(x=2) = \frac{{}^{20}\text{C}_2 \times {}^{30}\text{C}_3}{{}^{50}\text{C}_5} = 0,3641$ (8)

4.4 Let X = the number of green marbles. X ~ Bin $\left(n; \frac{1}{3}\right)$

$$\text{P}(x=0) = {}^{n}\text{C}_0 \times \left(\frac{1}{3}\right)^0 \times \left(\frac{2}{3}\right)^n = \left(\frac{2}{3}\right)^n$$

$$\therefore 1 - \left(\frac{2}{3}\right)^n > 0,975$$

$$\therefore \left(\frac{2}{3}\right)^n < 0,025$$

$\therefore$ n > 9,09

$\therefore$ n = 10 (10)

[36]

QUESTION 5

5.1 $\text{P}(\text{A}|\text{B}) = \frac{\text{P}(\text{A} \cap \text{B})}{\text{P}(\text{B})}$

$\therefore 0,1 = \frac{\text{P}(\text{A} \cap \text{B})}{0,3}$

$\therefore \text{P}(\text{A} \cap \text{B}) = 0,03$ (2)

5.2 (a) $\text{P}(\text{A} \cup \text{B}) = 0,5 + 0,3 - 0,03 = 0,77$ (2)

(b) P (1 event only) = P (A) + P (B) – 2P (A ∩ B)

or P (A ∪ B) – P (A ∩ B)

= 0,77 – 0,03

= 0,74 (4)

(c) $\text{P}(\text{B}|\text{A}) = \frac{0,03}{0,5} = 0,06$ (2)

5.3 P (A) . P(B) = 0,3 × 0,5 = 0,15

P (A ∩ B) = 0,03

$\therefore$ P (A) . P(B) ≠ P (A ∩ B)

$\therefore$ not independent. (3)

[13]

Total for Module 2: 100 marks

MODULE 3 FINANCE & MODELLING

QUESTION 1

1.1 $2393{,}43 = 1233{,}63a + 1125{,}50b$
$1233{,}63 = 1125{,}50a + 100b$
$\therefore a = 1{,}005$
$\therefore b = 1{,}025$ simultaneous method (7)

1.2 $T_1 \times 1{,}025 + 100 \times 1{,}005 = 1125{,}50$
$\therefore T_1 = 1000$ (3)
[10]

QUESTION 2

2.1 (a) Deposit is $0{,}1 \times 485\ 000 = 48\ 500$

$$i = \frac{0{,}11}{12}$$

$$\therefore 436\ 500 = x\left(\frac{1-(1+i)^{-240}}{i}\right)$$

$\therefore x =$ R4 505,50 (7)

(b) $$\text{BO} = 4505{,}50\left(\frac{1-(1+i)^{-204}}{i}\right) \quad (6)$$

= R415 107,83

$$415\,107{,}83 = x\left(\frac{1-(1+i)^{-204}}{i}\right) \text{ where } i = \frac{0{,}13}{12} \quad (6)$$

$\therefore x =$ R5 058,56 (12)

Alternative method:

$$\text{BO} = 436\ 500\,(1+i)^{36} - 4\ 505{,}50\left[\frac{(1+i)^{36}-1}{i}\right]$$

= 606 245,52 – 191 137,38
= R415 108,14 (then do last 2 steps).

2.2 (a) Interest = monthly withdrawal

$$\therefore 2\ 000\ 000 \times \frac{0.12}{12} = \text{R } 20\ 000 \quad (4)$$

(b) Total payments = $12 \times 28 + 1 = 337$

$$i = \frac{0{,}098}{12}$$

$$2\ 000\ 000 = x\,\frac{(1+i)^{337}-1}{i}$$

$\therefore x =$ R 1 126,26 (8)
[31]

QUESTION 3

3.1 Value of first 12 payments at 31/12/12 if $i = \frac{0,075}{12}$ is

$$\text{FV} = 250 \times \frac{(1+i)^{12} - 1}{i} \times (1+i)^{25} = 3628,70$$

Value of next 12 payments at 31/12/12 is

$$\text{FV} = 250 \times 1,1 \times \frac{(1+i)^{12} - 1}{i} \times (1+i)^{13} = 3\,704,02$$

Value of next 12 payments at 31/12/12 is

$$\text{FV} = 250 \times 1,1^2 \times \frac{(1+i)^{12} - 1}{i} \times (1+i)^{1} = 3\,780,90$$

Thus total expected investment is R 11 113,62 (10)

3.2 Number of payments missed is 5. Value of each payment is R 250.
Thus value of missed payments on 31/12/12 is

$$\text{V} = 250 \times \frac{\left((1+i)^5 - 1\right)}{i} \times (1+i)^{27} = \text{R}\,1\,497,61$$

Or the invest will only be worth: R 9 616,01 (8)

[18]

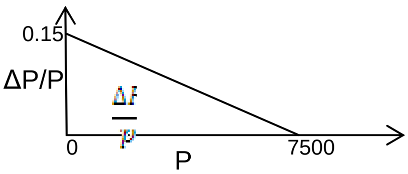
QUESTION 4

4.1 $\text{P}_{n+1} = \text{P}_n + 0,15\,\text{P}_n \left(1 - \frac{\text{P}_n}{7500}\right)$

$\text{P}_o = 10$

$\therefore \text{P}_{15} \approx 81$ elephants (iterations done on calculator) (8)

4.2



x-intercept = 7500 this is the carrying capacity
y-intercept = 0,15 this is the intrinsic growth rate

(5)

4.3 $\frac{\Delta \text{P}}{\text{P}} = -\frac{0,15}{7\,500}\text{P} + 0,15$ (4)

4.4 (a) $\frac{\Delta \text{P}}{\text{P}} = -\frac{0,15}{7\,500}3\,000 + 0,15 = 0,09$ or 9% per annum growth rate (3)

(b) $0,05 = -\frac{0,15}{7\,500}\text{P} + 0,15$

$\therefore$ P = 5 000 elephants. (3)

[23]

QUESTION 5

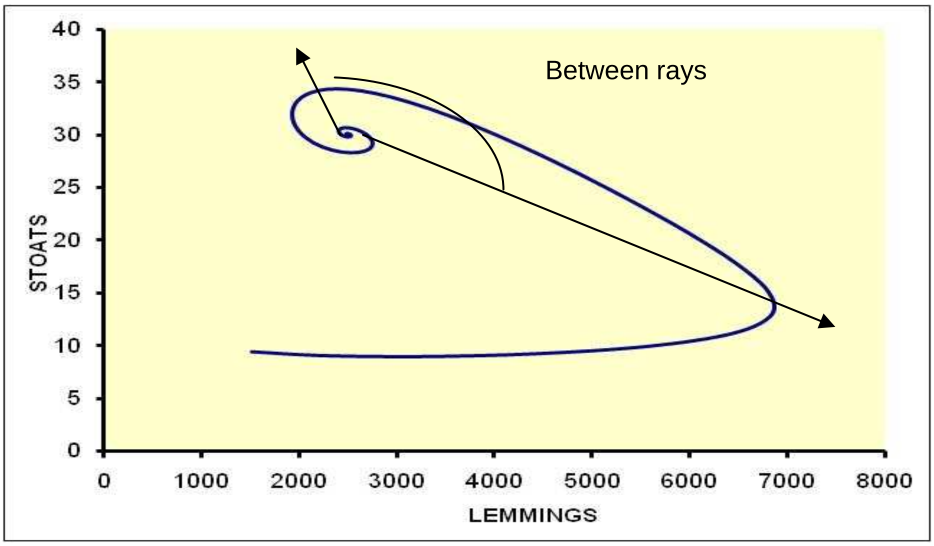
5.1 death rate = 10%
Therefore 1/10th of stoats die every year so their life span is 10 years. (3)

5.2 Intrinsic growth rate is 0,8 or 80% per annum. (1)

5.3 carrying capacity of the lemmings is 10 000. (1)

5.4 $L_0 \approx 1500$ and $S_0 \approx 10$ (2)

5.5



(2)

5.4 $S_n = S_n + 0{,}00006L_nS_n - 0{,}1S_n$
$\therefore L_n \approx 1\,667$ lemmings
$L_n = L_n + 0{,}8L_n\left(1 - \frac{L_n}{10\,000}\right) - 0{,}02L_nS_n$
$\therefore 0{,}02S_n = 0{,}8\left(1 - \frac{1\,667}{10\,000}\right)$
$\therefore S_n \approx 33$ stoats. (9)
[18]

Total for Module 3: 100 marks

MODULE 4 GRAPHS & MATRICES

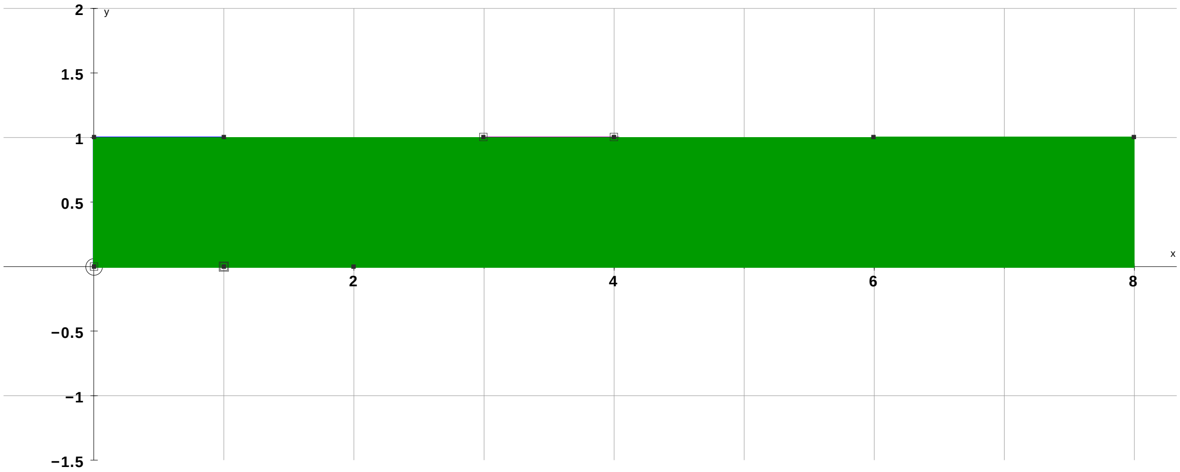
QUESTION 1

1.1 A onto B $\longrightarrow$ SP (4)
1.2 B onto C $\longrightarrow$ Q (2)
1.3 A onto D $\longrightarrow$ R^2 P (4)

[10]

QUESTION 2

2.1



(6)

2.2 No (2)

[8]

QUESTION 3

3.1 $\tan\theta = \frac{1}{\sqrt{3}} \quad \therefore \theta = 30°$ (2)

3.2 $$A \to B \begin{pmatrix} \cos 2\times 30° & \sin 2\times 30° \\ \sin 2\times 30° & -\cos 2\times 30° \end{pmatrix} = \begin{pmatrix} \cos 60° & \sin 60° \\ \sin 60° & -\cos 60° \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$$

(3)

3.3 $$B \to C \quad \text{Rotation through } 150° = \begin{pmatrix} \cos 150° & -\sin 150° \\ \sin 150° & \cos 150° \end{pmatrix} = \begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{-1}{2} \\ \frac{1}{2} & \frac{-\sqrt{3}}{2} \end{pmatrix}$$

(5)

[10]

QUESTION 4

4.1 $A = \begin{pmatrix} 1 & 2 & -1 \\ -1 & 1 & b \\ 2 & 3 & 1 \end{pmatrix}$

$\therefore \det A = 1(1-3b) - 2(-1-2b) - 1(-3-2)$

$= 1 - 3b + 2 + 4b + 3 + 2$

$= b + 8$

If no unique solution det A = 0

$\therefore\ b = -8$ (6)

4.2 (a) $A = \begin{pmatrix} 1 & 2 & -1 \\ -1 & 1 & 4 \\ 2 & 3 & 1 \end{pmatrix}$

$\therefore \det A = 1(1-12) - 2(-1-8) - 1(-3-2)$

$= -11 + 18 + 5$

$= 12$

Cofactors for $A = \begin{pmatrix} -11 & 9 & -5 \\ -5 & 3 & 1 \\ 9 & -3 & 3 \end{pmatrix}$

$A^{-1} = \frac{1}{12}\begin{pmatrix} -11 & -5 & 9 \\ 9 & 3 & -3 \\ -5 & 1 & 3 \end{pmatrix}$ (10)

4.2 (b) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{12}\begin{pmatrix} -11 & -5 & 9 \\ 9 & 3 & -3 \\ -5 & 1 & 3 \end{pmatrix}\begin{pmatrix} 8 \\ -5 \\ 11 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$ (4)

[20]

QUESTION 5

5.1 DF = 40
EF = 40
CD = 50
AB = 60
BC = 70 or AF = 70
AB = 60
Therefore minimum spanning tree requires 260m of cabling (8)

5.2 AB = 60
BC = 70
CD = 50
DF = 40
FE = 40
EA = 80
Therefore upper bound for shortest route is 340 m. (8)

[16]

QUESTION 6

6.1 Vertices D, E, F and G have odd degree
Paths: DE and FG = 10 + 8 = 18
DF and EG = 7 + 9 = 16
DG and EF = 15 + 17 = 32
Therefore duplicate edges: DF and BE and BG (8)

6.2 If GD is inserted with length 7 units.
Then only E and F have odd degree
The shortest path between EF is 17 units
Thus in Q 6.1 total length =sum all edges + 16
Now total length = sum all edges + 7 + 17
Thus new route would be 8 kms longer. (10)

6.3 Now only need to join vertices G and F with odd degree.
Shortest path between GF = 8 units
Thus total length = sum all edges + 8 = 77 + 8 = 85 km (6)

[24]

QUESTION 7

7. (a) all vertices degree 2
1 vertex degree 4 and 5 vertices degree 2.
(b) 2 vertices degree 4 and 4 vertices degree 2 – (× 2)
(c) 3 vertices degree 4 and 3 vertices degree 2
(d) 4 vertices degree 4 and 2 vertices degree 2
(e) 5 vertices degree 4 and 1 vertices degree 2
(f) 6 vertices degree 4
Can't have odd degree vertices as then no Eulerian circuit
Can't have degree 6 on 6 vertices without multiple edges
Can't have degree 0 if connected
Therefore 8 unique non-isomorphic Eulerian circuits. **[12]**

Total for Module 4: 100 marks

Total: 300 marks