GRADE 12 EXAMINATION
NOVEMBER 2011

ADVANCED PROGRAMME MATHEMATICS

MARKING GUIDELINES

Time: 3 hours | 300 marks

These marking guidelines were used as the basis for the official IEB marking session. They were prepared for use by examiners and sub-examiners, all of whom were required to attend a rigorous standardisation meeting to ensure that the guidelines were consistently and fairly interpreted and applied in the marking of candidates' scripts.

At standardisation meetings, decisions are taken regarding the allocation of marks in the interests of fairness to all candidates in the context of an entirely summative assessment.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines, and different interpretations of the application thereof. Hence, the specific mark allocations have been omitted.

MODULE 1 CALCULUS AND ALGEBRA

QUESTION 1

1.1 $p = 3ln3$ $q = 3ln6$

$= ln\ 27$ $= ln216$

$$\text{Gradient} = \frac{q - p}{6 - 3} = \frac{(ln216) - (ln27)}{3} = \frac{3ln6 - 3ln3}{3} = ln2$$

OR

$$q - p = 3ln6 - 3ln3 = 3ln6 - 3ln3 = 3ln2$$

$$\text{Gradient} = \frac{3ln2}{6 - 3} = ln2$$

(7)

1.2 $f(n+1) - 8\ f(n) = p \times 15n$

$\therefore \left[15^{n+1} - 8^{n+1-2}\right] - 8\left[15^{n} - 8^{n-2}\right] = p \times 15^{n}$

$\therefore 15^{n+1} - 8^{n-1} - 8.15^{n} + 8^{n-1} = p \times 15^{n}$

$\therefore 15^{n}(15 - 8) = p \times 15^{n}$

$\therefore 15^{n}(7) = p \times 15^{n}$

$\therefore p = 7$

(8)

[15]

QUESTION 2

2.1 (a) Assume (statement) true for $n = k$ (1)

(b) $$\text{LHS} = \left[1 + 8 + 27 + \cdots + k^{3}\right] + \left[(k+1)^{3}\right]$$

$$= \frac{k^{2}(k+1)^{2}}{4} + (k+1)^{3} \text{ from (5)}$$

$$= \frac{k^{2}(k+1)^{2} + 4(k+1)^{3}}{4}$$

$$= \frac{(k+1)^{2}\left[k^{2} + 4(k+1)\right]}{4}$$

$$= \frac{(k+1)^{2}\left[k^{2} + 4k + 4\right]}{4}$$

$$= \frac{(k+1)^{2}(k+2)^{2}}{4}$$

$= \text{RHS}$

OR

$$= \frac{k^{2}(k^{2} + 2k + 1) + 4(k^{3} + 3k^{2} + 3k + 1)}{4}$$

$$= \frac{k^{4} + 2k^{3} + k^{2} + 4k^{3} + 12k^{2} + 12k + 4}{4}$$

$$= \frac{k^{4} + 6k^{3} + 13k^{2} + 12k + 4}{4}$$

$$\text{RHS} = \frac{\left(k^{2} + 2k + 1\right)\left(k^{2} + 4k + 4\right)}{4}$$

$$= \frac{k^{4} + 6k^{3} + 13k^{2} + 12k + 4}{4}$$

$\therefore \text{LHS} = \text{RHS}$

(8)

2.2 $\Delta x = \dfrac{2-0}{n} = \dfrac{2}{n}$

$x_i = \dfrac{2i}{n}$

$f(x_i)\Delta x = \left(\dfrac{2i}{n}\right)^3 + 1 = \dfrac{8i}{n^3}^3 + 1$

$\text{Area} = \lim\limits_{n\to\infty} \dfrac{b-a}{n} \sum\limits_{i=1}^{n} f(x_i)$

$\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \left(\dfrac{8i^3}{n^3} + 1\right)$

$= \lim\limits_{n\to\infty} \dfrac{8}{n^3}\left[\dfrac{n^2(n^2 + 2n + 1)}{4}\right] + n$

$= \lim\limits_{n\to\infty} 2n + 4 + \dfrac{2}{n} + n$

$\text{Area} = \lim\limits_{n\to\infty} \dfrac{2}{n}\left(3n + 4 + \dfrac{2}{n}\right)$

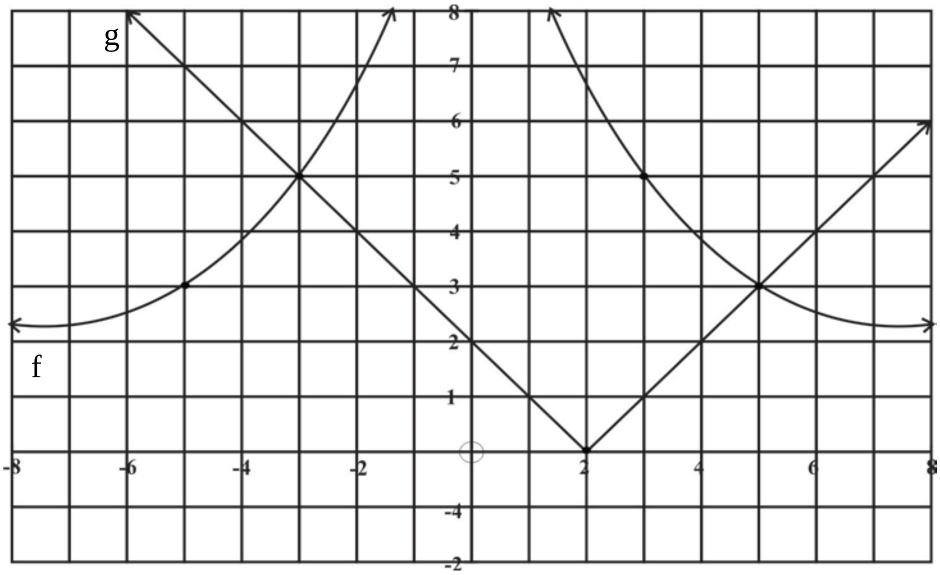
$= 6$

(11)

[20]

QUESTION 3

3.1



(7)

3.2 $x \in [-3;\ 0) \cup (0;5]$

or: $-3 \le x < 0$ or $0 < x \le 5$

or: $-3 \le x \le 5\,;\ \ x \ne 0$

(4)

[11]

QUESTION 4

4.1 Area segment AOB = $\frac{1}{2}r^2\theta$

Area Δ AOX = $\frac{1}{2}$ AO.OX $\sin\theta$

OX = $r\cos\theta$

$\therefore \Delta \text{AOX} = \frac{1}{2}r.r\cos\theta\sin\theta$

$= \frac{1}{2}r^2 \cos\theta\sin\theta$

Area AXB = $\frac{1}{2}r^2\theta - \frac{1}{2}r^2\cos\theta\sin\theta$

$= \frac{1}{2}r^2\ (\theta - \sin\theta\cos\theta)$ cm^2

OR

Area segment AOB = $\frac{1}{2}r^2\theta$

Area Δ AOX = $\frac{1}{2}$ OX.XA

XA = $r\sin\theta$

Area Δ AOX = $\frac{1}{2}r\cos\theta\ r\sin\theta$

$= \frac{1}{2}r^2\cos\theta\sin\theta$

$\therefore$ Area AXB = $\frac{1}{2}r^2\theta - \frac{1}{2}r^2\cos\theta\sin\theta$

$= \frac{1}{2}r^2\ (\theta - \sin\theta\cos\theta)$ cm^2

(7)

4.2 Perim AXB

$= \text{AX} + \text{AB} + \text{XB}$

$= r\sin\theta + r\theta + (r - r\cos\theta)$

$= 12\sin\frac{\pi}{6} + 12\left(\frac{\pi}{6}\right) + 12 - 12\cos\frac{\pi}{6}$ (subst)

$= 6 + 2\pi + 12 - 6\sqrt{3}$

$= 18 + 2\pi - 6\sqrt{3}$ cm

(8)

[15]

QUESTION 5

5.1 $R_1 = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$; $R_3 = -1 + 2\sqrt{3}$

$\therefore R_2 = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$ $\therefore R_4 = -1 - 2\sqrt{3}$

$(SR)_1 = -1$ $(SR)_2 = -2$

$(PR)_1 = \frac{1}{4} - \frac{3}{4}i^2$ $(PR)_2 = 1 - 12$

$= \frac{1}{4} + \frac{3}{4}$ $= -11$

$= 1$

$\therefore$ 2 quadratic factors $= \left(x^2 - SR_1\ x + PR_1\right)\left(x^2 - SR_2 x + PR_2\right)$

$= \left(x^2 + x + 1\right)(x^2 + 2x - 11)$

$\therefore P(x) = \left(x^2 + x + 1\right)\left(x^2 + 2x - 11\right)\left(x + 2\right)$ (14)

OR

$\left(x + \frac{1}{2}\right)^2 = \left(\frac{\pm\sqrt{3}}{2}i\right)^2$

$x^2 + x + \frac{1}{4} = \frac{3}{4}i^2$

$x^2 + x + 1 = 0$ and

$\left(x + 1\right)^2 = \left(\pm 2\sqrt{3}\right)^2$

$x^2 + 2x + 1 = 12$

$x^2 - 2x - 11 = 0$

$\therefore P(x) = \left(x^2 + x + 1\right)\left(x^2 + 2x - 11\right)\left(x + 2\right)$

5.2 Real roots: $x = -1 \pm 2\sqrt{3}$; $x = -2$ (2)

[16]

QUESTION 6

6.1 (a) $x = 3$ (2)

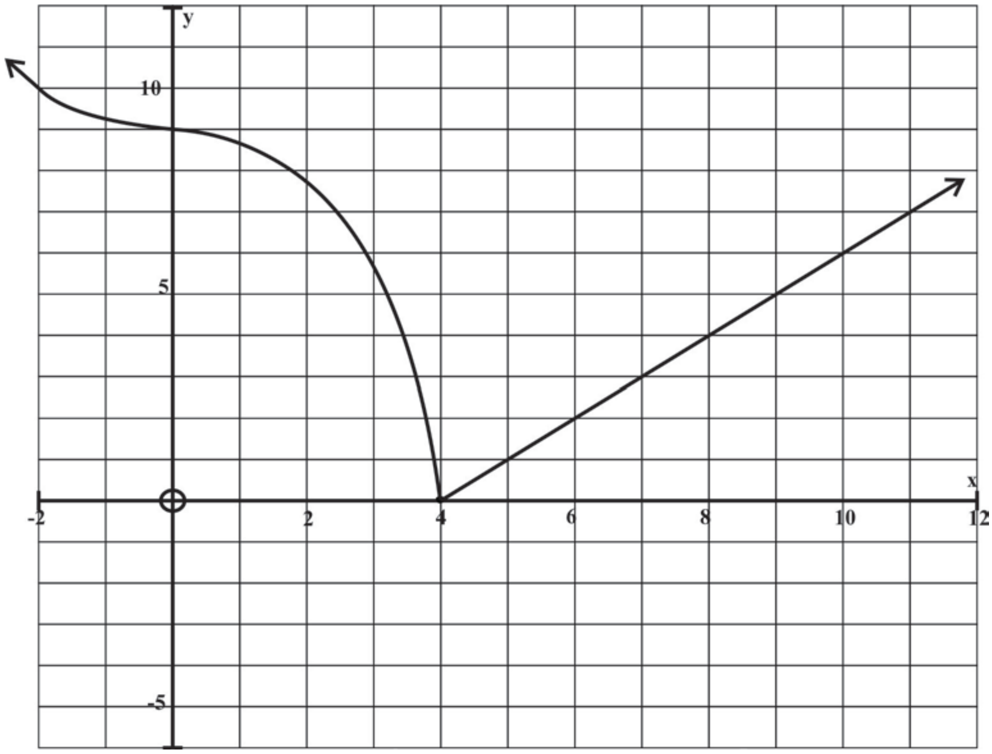
(b) $x = -1$ (2)

(c) $x = 1$ (2)

(d) $x = -2$; $x > 3$ (3)

(e) $x < -1$ (graph concave up) (3)

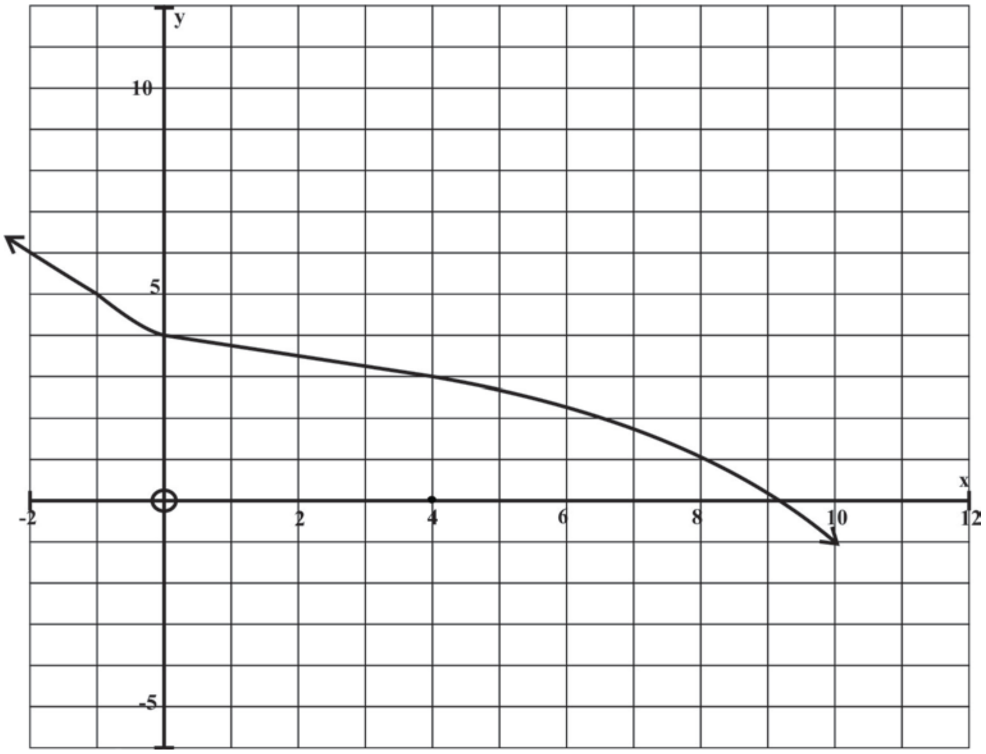
6.2 (a)



(3)

(b)

$F^{-1}(x)$



(5)

[20]

QUESTION 7

7.1 $y = x^{\frac{1}{3}} \cos\left(x^2 - 1\right)$

$$\frac{dy}{dx} = \frac{1}{3} x^{-\frac{2}{3}} . \cos\left(x^2 - 1\right) + x^{\frac{1}{3}} . \sin\left(x^2 - 1\right).2x$$

(9)

7.2 (a) $f(x) = \cot x$

$f'(x) = -\text{cosec}^2 x$ (1)

$f''(x) = (-2\text{cosec}x)(.-\cot x\,\text{cosec}x)$ (3)

(b) $f'\left(\frac{\pi}{2}\right) = -\left(\text{cosec}2\,\frac{\pi}{2}\right) = -1$

$\therefore\ f'\left(\frac{\pi}{2}\right) \neq 0$ (2)

$f''\left(\frac{\pi}{2}\right) = \left(-2\,\text{cosec} - \frac{\pi}{2}\right)\left(\pi \cot\frac{\pi}{2}.\text{cosec}\frac{\pi}{2}\right)$

$= -2(1)\,(0)$

$= 0$ (3)

$\therefore$ Non stat point of inflection when $x = \dfrac{\pi}{2}$

7.3 $2 + 3y^1 - \left(y^2 + 2xy\,y^1\right) = 0$

$y^1\left(3 - 2xy\right) = y^2 - 2$

$y^1 = \dfrac{y^2 - 2}{(3 - 2xy)}$

$y^1\ (-1;-2) = \dfrac{(-2)^2 - 2}{3 - 2\left(-1\right)(-2)}$

$= \dfrac{2}{3-4}$

$= -2$

OR

$2 + 3\dfrac{dy}{dx} - y^2 - 2xy.\dfrac{dy}{dx} = 0$

$\dfrac{dy}{dx}\left(3 - 2xy\right) = y^2 - 2$

$\dfrac{dy}{dx} = \dfrac{y^2 - 2}{3 - 2xy}$

$\therefore$ when $x = -1;\ y = -2$

$\dfrac{dy}{dx} = \dfrac{\left(-2\right)^2 - 2}{3 - 2\left(-1\right)\left(-2\right)}$

$= \dfrac{2}{3-4}$

$= -2$

$\therefore\ y - y_1 = m(x - x_1)$

$\therefore\ y + 2 = -2(x+1)$

$\therefore\ y = -2x - 4$

or $y = mx + c$

$-2 = -2(-1) + c$

$c = -4$

$\therefore\ y = -2x - 4$ (13)

[31]

QUESTION 8

8.1 (a) $f(x) = \dfrac{3(x-3)}{(x-2)(x+1)}$

Vert. Asympt. $x = 2;\ x = -1$

Horiz. Asympt. $y = 0$ (5)

(b) At A, $f'(x) = 0$

$\therefore (x^2 - x - 2)(3) - (3x - 9)(2x - 1) = 0$ $\qquad N = 0;\ D \neq 0$

$\therefore 3x^2 - 3x - 6 - 6x^2 + 21x - 9 = 0$

$\therefore 3x^2 + 18x - 15 = 0$

$\therefore x^2 - 6x + 5 = 0$

$(x-5)(x-1) = 0$

$x = 5 \qquad x = 1$

N/A (10)

(c) Since A is to left of $x = 3$ (1)

8.2 $p(x) = \tan\left(x - \dfrac{\pi}{4}\right) - \dfrac{x}{2}$

$p^1(x) = \sec^2\left(x - \dfrac{\pi}{4}\right) - \dfrac{1}{2}$

$a_{n+1} = a_n - \dfrac{p(a_n)}{p^1(a_n)}$

$a_1 = \dfrac{\pi}{2}$

$a_2 = \dfrac{\pi}{2} - \left[\dfrac{\tan\dfrac{\pi}{4} - \dfrac{\pi}{8}}{\sec^2\left(\dfrac{\pi}{4}\right) - \dfrac{1}{2}}\right]$ $\qquad \tan\dfrac{\pi}{4} = 1$

$= 1{,}1659$ $\qquad \sec\dfrac{\pi}{4} = \sqrt{2}$

(6)

8.3 $c(t) = t(1-t)^{\frac{1}{2}}$

$c^1(t) = (1-t)^{\frac{1}{2}} + t.\frac{1}{2}(1-t)^{-\frac{1}{2}}(-1) = 0$

$2(1-t)^{\frac{1}{2}} = \dfrac{t}{(1-t)^{\frac{1}{2}}}$

$2(1-t) = t$

$2 - 2t = t$

$t = \frac{2}{3}$

OR $\therefore$ month $\frac{2}{3}(12) = \text{August}$

(9)

[31]

QUESTION 9

9.1 (a) Let $u = 5x - 1$

$du = 5dx$

$\frac{1}{5}\,du = dx$

$\therefore$ Int $= \frac{1}{5}\int 2(u)^{\frac{1}{2}}\,du$

$= \frac{2}{5}(2)\left(\sqrt{5x-1}\right) + c$

OR

$\int 2(5x-1)^{-\frac{1}{2}}\,dx$

$= \dfrac{2(5x-1)^{\frac{1}{2}}}{\frac{1}{2}.5} + c$

(5)

(b) $u = 5x^2 - 1$

$du = 10x\,du$

$\frac{1}{5}\,du = 2x\,dx$

$\therefore$ Int $= \frac{1}{5}\int u^{-\frac{1}{2}}\,du$

$= \frac{2}{5}\left(5x^2 - 1\right)^{\frac{1}{2}} + c$

OR

$\int 2x(5x^2-1)^{-\frac{1}{2}}\,dx$

$= \dfrac{2(5x^2-1)^{\frac{1}{2}}}{5} + c$

(6)

(c) $\int fg^1\,dx = \quad fg - \int f^1 g\,dx$

Let $f = 2x \qquad g^1 = \dfrac{1}{\sqrt{5x-1}}\ dx$

$f^1 = 2dx$

$g = (5x-1)^{-\frac{1}{2}}\ dx$

$\dfrac{(5x-1)^{\frac{1}{2}}}{5.\frac{1}{2}}$

$\therefore \text{Int} = \dfrac{2x.(5x-1)^{\frac{1}{2}}}{5.\frac{1}{2}} - 4\int \dfrac{(5x-1)^{\frac{1}{2}}}{5}\,dx$

$\dfrac{4x(5x-1)^{\frac{1}{2}}}{5} - \dfrac{4}{5}\dfrac{(5x-1)^{\frac{3}{2}}}{\frac{3}{2}} + c$ (9)

9.2 (a) $\dfrac{1}{x^2(x+1)} = \dfrac{a}{x} + \dfrac{b}{x^2} + \dfrac{c}{(x+1)}$

$\therefore 1 = ax(x+1) + b(x+1) + cx^2$ OR (by equating coefficients)

When $x = 0$: $1 = b$

When $x = -1$ $1 = c$

When $x = 1$ $1 = a(2) + b(2) + c$ $\quad x^2 : 0 = a + c$

$\therefore 1 = 2a + 2 + 1$ $\quad \therefore a = -1$

$a = -1$

$\therefore \dfrac{1}{x^2(x+1)} = \dfrac{-1}{x} + \dfrac{1}{x^2} + \dfrac{1}{x+1}$ (6)

(b) $\therefore \int \dfrac{1}{x^3 + x^2}\ dx = -\int \dfrac{1}{x}\,dx + \int \dfrac{1}{x^2}\,dx + \int \dfrac{1}{x+1}\,dx$

$= ln|x| + \dfrac{x^{-1}}{-1} + ln\,|x+1| + c$

$= ln\left|\dfrac{x+1}{x}\right| - x^{-1} + c$ (5)

[31]

QUESTION 10

10.1 LHS $= \sin^2 x + 2\sin x\cos x + \cos^2 x$

$= 1 + 2\sin x\ \cos x$ (2)

10.2 $\text{v} = \pi \int_{o}^{a} y^2 \, dx$

$$\therefore \pi \int_{o}^{a} \left(1 + \sin 2x\right) dx = \pi\left(a + 1\right)$$

$$\therefore \left. \left(x - \frac{1}{2}\cos 2x \right) \right]_{o}^{a} = \left(a + 1\right)$$

$$\therefore \left(a - \frac{1}{2}\cos 2a \right) - \left(-\frac{1}{2}\cos 0 \right) = a + 1$$

$$\therefore a - \frac{1}{2}\cos 2a + \frac{1}{2} = a + 1$$

$$\therefore \quad \left. \begin{aligned} -\frac{1}{2}\cos 2a &= \frac{1}{2} \\ \cos 2a &= -1 \end{aligned} \right\}$$

$$2a = \pi$$

$$a = \frac{\pi}{2}$$

(8)

[10]

Total for Module 1: 200 marks

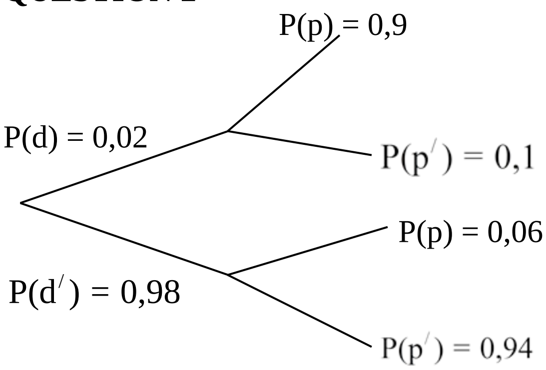
MODULE 2

QUESTION 1

$$\binom{10}{8}(0,7)^8(0,3)^2$$
$$= 0,2335$$

[9]

QUESTION 2



$$P(d|p)$$
$$= \frac{P(d\cap p)}{[illegible]}$$

Alternate
Assume a sample of 10 000 people.

10 000			
Has disease = 200		No disease = 9 800	
+ Test 180	– Test 20	+ Test 588	– Test 9 212

Number who have disease and + test $= 180$
Total with + test = 180 + 588 $= 768$
$\therefore$ required probability $= \frac{180}{768}$
$= 0,2344$

[10]

QUESTION 3

3.1 Strong negative correlation. The colder the temperature, the more cups of chocolate will be sold. (3)

3.2 No correlation between the packets of chips sold and the temperature. (2)

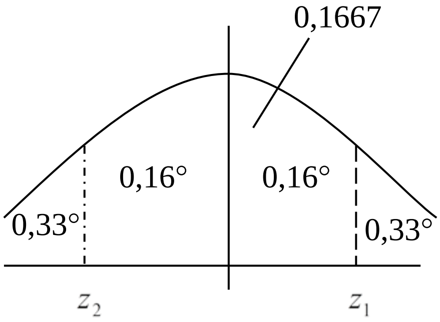
3.3 (a) $y_1 = 413 - 11{,}22(9)$
$= 312$ (2)

(b) $y_2 = 202 + 1{,}16(21)$
$= 226$ or correlation too low to make a prediction (2)

3.4 y_1 is reliable: even though extrapolation is used, the temperature is close to the data range given and the correlation coefficient is high.
y_2 is not reliable: there is no correlation.
Both models are limited if the temperature is extrapolated too low or too high. (4)

[13]

QUESTION 4



$\mu = 134$
$\sigma = 16$
$z_1 = 0,44$
$0,43 = \dfrac{x-134}{16}$
$16(0,43) + 134 = x$
$x = 140,88$

$z_2 = -0,44$
$-0,43 = \dfrac{x-134}{16}$
$x = 16(-0,44) + 134$
$x = 127,12$

Medium – between 127 and 141 minutes

[10]

QUESTION 5

5.1 $\dfrac{\binom{13}{5} \times 4}{\binom{52}{5}} = 0,00198$ **or** $\dfrac{13}{52} \times \dfrac{12}{51} \times \dfrac{11}{50} \times \dfrac{9}{49} \times \dfrac{8}{48} \times 4$ (8)

5.2 $\dfrac{\binom{4}{3}\binom{4}{2} \times 13 \times 12}{\binom{52}{5}} = 0,00144$ (8)

[16]

QUESTION 6

6.1 $\int_0^3 (3kx^2 - kx^3)dx = 1$

$$\left[kx^3 - \frac{k}{4}x^4\right]_0^3 = 1$$

$$27k - \frac{81}{4}k = 1$$

$$108k - 81k = 4$$

$$27k = 4$$

$$k = \frac{4}{27}$$ (8)

6.2 $f(x) = \frac{4}{9}x^2 - \frac{4}{27}x^3$

$$f'(x) = \frac{8}{9}x - \frac{4}{9}x^2$$

$$8x - 4x^2 = 0$$

$$2x - x^2 = 0$$

$$x(x-2) = 0$$

$$x = 2$$

Mode is 2 (8)

[16]

QUESTION 7

7.1 Let μ_1 be the mean life time of 'Dura Power' batteries

Let μ_2 be the mean life time of 'Long Life' batteries

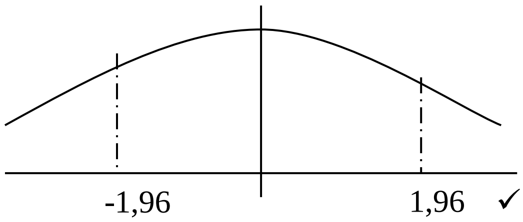
$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$

$\sigma_1 = 20 \quad n_1 = 100 \quad \sigma_2 = 19 \quad n_2 = 100$

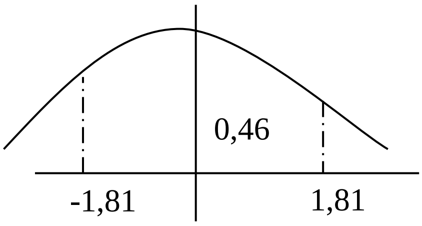
$$z = \frac{58{,}5 - 53{,}5}{\sqrt{\frac{400}{100} + \frac{361}{100}}}$$

$= 1{,}81$



There is insufficient evidence at the 5% significance level to reject the null hypothesis. Insufficient evidence of a difference in the mean lifetimes of the two makes of batteries. (11)

7.2



Z > 1,81
0,5-0,4649
= 0,0351
Two tailed 7,02% sig. level

(6)

[17]

QUESTION 8

$$\binom{10}{5}\binom{5}{4}\binom{1}{1} + \binom{10}{4}\binom{6}{4}\binom{2}{2} + \binom{10}{5}\binom{5}{3}\binom{2}{2}$$

= 6930

[9]

Total for Module 2: 100 marks

MODULE 3 FINANCES AND MODELLING

QUESTION 1

1.1 $Fv = \dfrac{3200\left[\left(1+\dfrac{0,074}{12}\right)^{30}-1\right]}{\dfrac{0,074}{12}}$ **= R105 099,30** (6)

1.2 $105000 = \dfrac{x\left(1+\dfrac{0,074}{12}\right)\left[\left(1+\dfrac{0,074}{12}\right)^{30}-1\right]}{\dfrac{0,074}{12}}$

x = R 3 177,38 (8)

[14]

QUESTION 2

2.1 $150000\left(1+\dfrac{0,0625}{12}\right)^{3}$ = **R152 355,98** (4)

2.2 152 355,98 – 150 000 = R2 355,98

3 400 – 2 355,98 = **R1 044,02** (4)

2.3 $T_n = T_{n-1}\left(1+\dfrac{0,0625}{12}\right) - 3400$

$T_0 = 150000\left(1+\dfrac{0,0625}{12}\right)^{2} = 151566,57$

T_{46} = 16 268,76

OR

$150000\left(1+\dfrac{0,0625}{12}\right)^{48} - \dfrac{3400\left[\left(1+\dfrac{0,0625}{12}\right)^{46}-1\right]}{\dfrac{0,0625}{12}}$

= 192 478,89 – 176 210,13 **= 16 268,76** (10)

2.4 $10500\left(1+\dfrac{0,0625}{12}\right)^{6} + 5000\left(1+\dfrac{0,0625}{12}\right)^{4}$ = **15 937,41**

15 937,41 < 16 268,76 **will not be in overdraft**. (10)

[28]

QUESTION 3

$$36\ 000 = \frac{4\ 000\,[1-(1+0,082)^{-n}]}{0,082}$$

$1,082^{-n} = 0,262$

$n = 16,995 \approx$ **17 payments**

[8]

QUESTION 4

4.1 (a) Logistic model : a carrying capacity exists (2)

(b) Phase 1 : greatest change on y-axis exists (2)

(c) population nears carrying capacity , hence starts stabilising (2)

4.2 (a) $L_{n+1} = L_n + 0,104.L_n\left(1-\frac{L_n}{20}\right)-1 \quad ,L_0 = 8$

L₆ = 4,75 **population is dying out.** (8)

(b) $L_{n+1} = L_n + 0,104.L_n\left(1-\frac{L_n}{20}\right)-0,5 \quad ,L_0 = 8$

L₆ = 7,995 **population is stable**. (4)

4.3 I = birth rate – death rate = 1 × 3 × 0,5 × survival – 1/6 = 0,104

Survival = 18% (6)

[24]

QUESTION 5

5.1 (a) equilibrium : flattens out (2)

(b) 70 – 80 years (2)

(c) not a good idea to cull:
long term projection is that at 200 years there are very few seals.
culling could lead to long term extinction

OR

good idea to cull:
for next century no real impact on population
century is a long time: much could happen to increase numbers again (2)

(d) not realistic because at equilibrium there are more sharks than seals

OR

might be realistic since seals are not the exclusive diet of sharks;
not a contained environment (2)

5.2 (a) $C = \dfrac{1}{0,033\,333}$ $\therefore$ lifespan $= 30$ years (2)

(b) number of seals killed per birth cycle. (2)

(c) rate of deadly interaction for seals by sharks. (2)

(d) 33 000 = 30 000 + 0,38 × 30 000 (1 – 30 000 / 44 000) – b × 30 000 × 30

b = 0,0007 (6)

[20]

QUESTION 6

6. 6.1 $x^2 - x - 1 = 0$ (2)

6.2 $x_1 = 1,61803$
$x_2 = -0,61803$
ratio tends to 1,61803 (4)

[6]

Total for Module 3: 100 marks

MODULE 4 MATRICES AND GRAPH THEORY

QUESTION 1

1.1 (a) reflection in the line y = x (2)

(b) stretch of factor of –3 , parallel to x-axis (y-axis invariant)

OR

Reflection in the y-axis followed by stretch factor 3, parallel to x-axis (y-axis invariant) (4)

1.2 (a) $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ (2)

(b) $\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$ shear of factor of 2 , parallel to y-axis (4)

[12]

QUESTION 2

2.1 $360^0 \div 5 = 72^0$ (2)

2.2 $\begin{pmatrix} \cos 72 & -\sin 72 \\ \sin 72 & cos 72 \end{pmatrix} \begin{pmatrix} 26,4 \\ 19,1 \end{pmatrix} = \begin{pmatrix} -10 \\ 31 \end{pmatrix}$ **E(–10; 31)** (6)

2.3 $\begin{pmatrix} \sqrt{k} & 0 \\ 0 & \sqrt{k} \end{pmatrix}$ (2)

2.4 $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -cos 2\theta \end{pmatrix} \begin{pmatrix} 26,4 & 26,4 \\ 19,1 & -19,1 \end{pmatrix} = \begin{pmatrix} 31 & 19,2 \\ -10 & 26,3 \end{pmatrix}$

26,4.cos 2θ + 19,1.sin 2θ = 31 OR 26,4.cos2θ – 19,1.sin 2θ = 19,2
–19,1.cos 2θ + 26,4.sin 2θ = –10 OR 19,1.cos2θ + 26,4.sin 2θ = 26,3

Cos2θ = 0,950 … and sin 2θ = 0,308 …
$\theta = 9^0$

OR

$\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -cos 2\theta \end{pmatrix} = \begin{pmatrix} 31 & 19,2 \\ -10 & 26,3 \end{pmatrix} \begin{pmatrix} 26,4 & 26,4 \\ 19,1 & -19,1 \end{pmatrix}^{-1}$

$\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -cos 2\theta \end{pmatrix} = -0,000991 \begin{pmatrix} 31 & 19,2 \\ -10 & 26,3 \end{pmatrix} \begin{pmatrix} -19,1 & -26,4 \\ -19,1 & 26,4 \end{pmatrix}$

$\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -cos 2\theta \end{pmatrix} = \begin{pmatrix} 0,950 & 0,309 \\ 0,309 & -0,950 \end{pmatrix}$

$\therefore \cos 2\theta = 0,950$

$\therefore \theta = 9°$ (12)

[22]

QUESTION 3

3.1 det P = –1(12 – 2) – 1(6 + 2) + 5(4 + 8) = 42

$$\begin{pmatrix} 10 & 8 & 12 \\ -7 & 7 & 0 \\ -19 & -11 & -6 \end{pmatrix} \rightarrow \begin{pmatrix} 10 & -8 & 12 \\ 7 & 7 & 0 \\ -19 & 11 & -6 \end{pmatrix} \rightarrow \begin{pmatrix} 10 & 7 & -19 \\ -8 & 7 & 11 \\ 12 & 0 & -6 \end{pmatrix}$$

$$\frac{1}{42}\begin{pmatrix} 10 & 7 & -19 \\ -8 & 7 & 11 \\ 12 & 0 & -6 \end{pmatrix}$$ (10)

3.2

$$\begin{array}{cccc} -2 & 0 & 1 & 5 \\ 1 & -1 & -3 & -15 \\ 1 & 1 & -2 & 6 \\ \hline -2 & 0 & 1 & 5 \\ 0 & -2 & -5 & -25 \\ 0 & 2 & -3 & 17 \\ \hline -2 & 0 & 1 & 5 \\ 0 & -2 & -5 & -25 \\ 0 & 0 & -8 & -8 \\ \hline \end{array}$$

–8z = –8 z = 1

–2y – 5(1) = –25 y = 10

–2x + (1) = 5 x = –2

OR

$$\begin{array}{cccc} 1 & -1 & -3 & -15 \\ 1 & 1 & -2 & 6 \\ -2 & 0 & 1 & 5 \\ \hline 1 & -1 & -3 & -15 \\ 0 & -2 & -1 & -21 \\ 0 & -2 & -5 & -25 \\ \hline 1 & -1 & -3 & -15 \\ 0 & -2 & -1 & -21 \\ 0 & 0 & 4 & 4 \\ \hline \end{array}$$

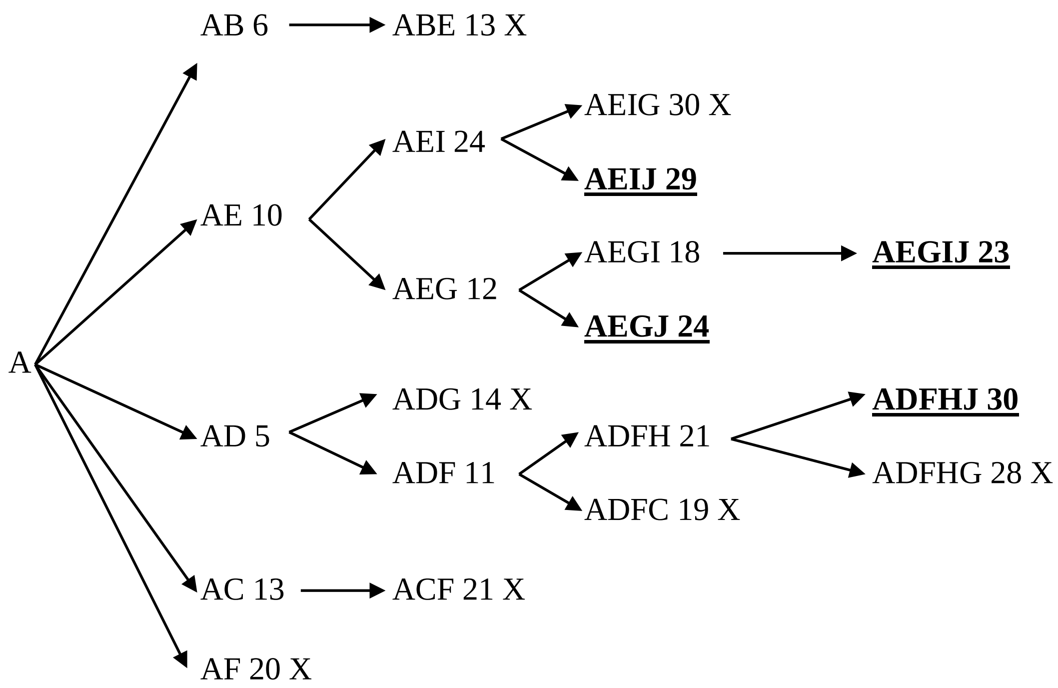
4z = 4 z = 1

–2y – (1) = –21 y = 10

1x – (10) – 3(1) – 15 x = –2 (10)

[20]

QUESTION 4



(phase 1) (phase 2) (phase 3) (solution)

[10]

QUESTION 5

5.1

CG	65	CG	65	CG	65
JD	90	CP	100	JG	100
JG	100	GJ	100	JD	90
CP	100	JD	90	CP	100
JW	120	WJ	120	WJ	120
+	155	+	155	+	155
	630		630		630

(8)

5.2

CG	65
CB	95
JB	70
JD	90
DJP	90 + 110
PJW	110 + 120
WCG	135 + 65
	950 km

(8)

5.3 No , length of second half is substantially greater than first half. (2)

5.4

900	G	J	P	C	B	D	J	W	C	G
880	G	C	W	J	B	D	C	P	C	G

Any route using all vertices, where $630 < \times < 950$ (8)

distance circuit using all seven vertices

– for each vertex left out

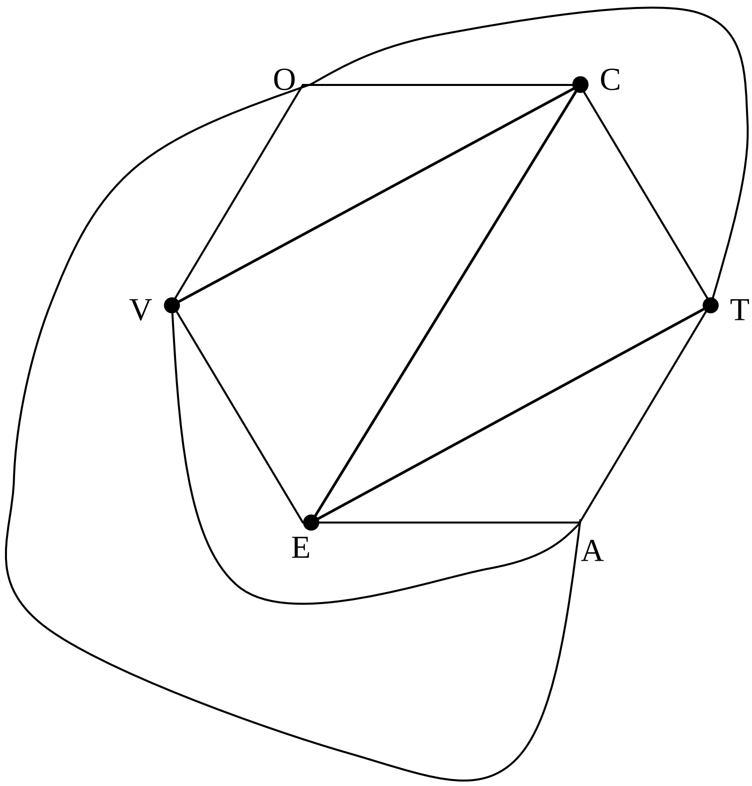
[26]

QUESTION 6

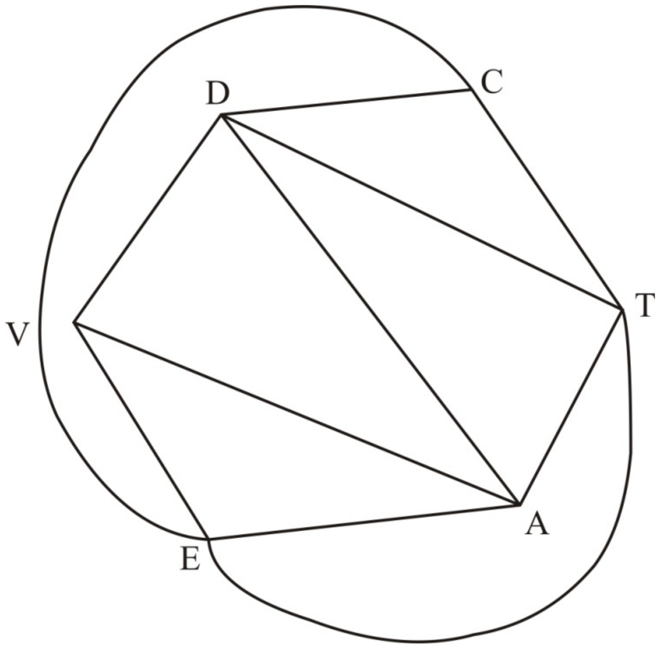
6.1 yes , all vertices have the same degree. (2)

6.2 yes , all vertices have even degrees. (2)

6.3



OR



(6)
[10]

Total for Module 4: 100 marks

Total: 300 marks