



GRADE 12 EXAMINATION
NOVEMBER 2017

ADVANCED PROGRAMME MATHEMATICS: PAPER II

MARKING GUIDELINES

Time: 1 hour

100 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

MODULE 2 STATISTICS

QUESTION 1

1.1
$$\frac{\binom{5}{3}\binom{7}{2}}{\binom{12}{5}} = 0.2652 \quad (6)$$

1.2
$$P(X = 7) = \binom{10}{7}(0.7)^7 (0.3)^3 = 0.2668 \quad (5)$$

1.3 (a)
$$\frac{5!}{2!2!} + 2 \times \frac{5!}{2!3!} = 50 \quad (6)$$

(b)
$$5 \times 2 + 2 = 12 \quad \{(CHLHL) \times 5 + (HLCLH) \times 2 = 12\} \quad (6)$$

[23]

QUESTION 2

2.1 (a)
$$0,3 \times (0,7) + (0,3)(0,7)^2 + (0,3)(0,7)^3 + (0,3)(0,7)^4 + C = 1$$

$$C = 0,4681 \quad (6)$$

(b)
$$P(X > 3) = P(X = 4) + P(X = 5)$$

$$= 0,3 (0,7)^4 + 0,4681$$

$$= 0,5401 \quad (3)$$

2.2 (a) A (1)

(b) (i)
$$\frac{n}{500} = \frac{0,2278 + 0,2922}{2}$$

$$\therefore n = 130 \quad (4)$$

(ii)
$$0,26 + Z\sqrt{\frac{(0,26)(0,74)}{500}} = 0,2922$$

$$Z = 1,64$$

$$\therefore \alpha = 90 \quad (6)$$

[20]

QUESTION 3

3.1 $X \sim N(9, 0,1^2)$

$$\begin{aligned}
 P(X > 8,9) &= P\left(Z > \frac{8,9-9}{0,1}\right) \\
 &= P(Z > -1) \\
 &= 0,5 + 0,3413 \\
 &= 0,8413
 \end{aligned}$$

(6)

3.2 $X \sim B(6, 0,8413)$

$$\begin{aligned}
 P(X \geq 2) &= 1 - \left[\binom{6}{0}(0,8413)^0(0,1587)^6 + \binom{6}{1}(0,8413)^1(0,1587)^5 \right] \\
 &= 0,9995
 \end{aligned}$$

(8)

3.3 $P(X < a) = 0,04$

$$-1,75 = \frac{a-9}{0,1}$$

$$\therefore a = 8,825 \text{ cm}$$

(6)
[20]

QUESTION 4

4.1 (a) $1 + m^2 + (m + 1)^2 + 4^2 + 5^2 = 55$

$$2m^2 + 2m - 12 = 0$$

$$m^2 + m - 6 = 0$$

$$(m + 3)(m - 2) = 0$$

$$m \neq -3 \text{ or } m = 2$$

$$\frac{5+t-1+4+3+t}{5} = 3$$

$$2t + 11 = 15$$

$$2t = 4$$

$$t = 2$$

(8)

(b) $r = -0,4$

(1)

(c) (i) $y = 4,2 - 0,4x$

(3)

(ii) $y = 4,2 - 0,4(6)$

$$y = 1,8$$

This is an unreliable estimation as the correlation is weak.

(3)

4.2 $H_0 : \mu = 49,5$

$H_1 : \mu < 49,5$

Rejection Region

Reject H_0 if $Z < -1,48$

Test Statistic:

$$Z = \frac{48 - 49,5}{\frac{4,8}{\sqrt{40}}} = -1,98$$

Conclusion: since $Z < -1,48$, we reject H_0 at the 7% l.o.s. and suggest sufficient evidence to support the claim that Basi's working hours per week have decreased significantly.

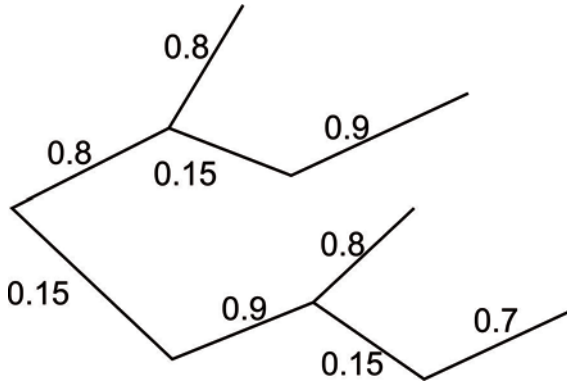
(10)
[25]

QUESTION 5

5.1 $0,8 + 0,15(0,9) = 0,935$

(4)

5.2



$(0.8)(0.8) + (0.8)(0.15)(0.9) + (0.15)(0.9)(0.8) + (0.15)(0.9)(0.15)(0.7) = 0.8702$

(8)
[12]

Total for Module 2: 100 marks

MODULE 3 FINANCE AND MODELLING

QUESTION 1

- 1.1 R1 200 000 ✓ (1)
 - 1.2 50 ✓ (1)
 - 1.3 ± R1 100 000 ✓✓ method: 2,3 – 1,2 = 1,1 million (2)
 - 1.4 ± R900 000 ✓✓ (2)
 - 1.5 (a) straight line gradient ✓ unaffected ✓ (2)
 - (b) interest ✓ has been increased ✓
- OR**
- Withdrawal ✓ from loan ✓ (2)

[10]

QUESTION 2

2.1
$$P \left(1 + \frac{0,0568}{12} \right)^2 \checkmark = \frac{5154,26 \checkmark \left[1 - \left(1 + \frac{0,0568}{12} \right)^{-34} \checkmark \checkmark \right]}{\frac{0,0568}{12}}$$

P = R160 000 ✓✓ (6)

2.2
$$OB = \frac{5154,26 \checkmark \left[1 - \left(1 + \frac{0,0568}{12} \right)^{-12} \checkmark \right]}{\frac{0,0568}{12}}$$

OB = R59 989,47 ✓✓ (5)

- 2.3 Jude paid: $5\ 154,26 \times 12 \checkmark = 61\ 851,12 \checkmark$
- Balance decreased: $116\ 674,09 - 59\ 989,47 \checkmark = 56\ 684,62 \checkmark$
- Interest paid: $61\ 851,12 - 56\ 684,62 = \mathbf{R5\ 166,50} \checkmark$ (5)

[16]

QUESTION 3

$$3.1 \quad 2\,000\,000 = A(1 + 0,068)^8 \checkmark\checkmark \quad \mathbf{A = 1\,181\,571,41 \checkmark\checkmark} \quad (4)$$

$$3.2 \quad 1\,181\,571,41 = 3\,400\,000(1 - i)^8 \checkmark\checkmark \quad \mathbf{i = 12,38\% \checkmark\checkmark} \quad (4)$$

$$3.3 \quad \left(1 + \frac{0,0764}{12}\right)^{12} \checkmark\checkmark = \left(1 + \frac{i}{2}\right)^2 \checkmark\checkmark \quad \mathbf{i = 7,7626\% \checkmark\checkmark}$$

$$5\,500\,000 \checkmark + \checkmark 300\,000 \checkmark \left(1 + \frac{0,077626}{2}\right)^6 \checkmark = 5\,877\,003,11 \checkmark$$

$$x \frac{\left[\left(1 + \frac{0,077626}{2}\right)^9 \checkmark - 1\right] \left(1 + \frac{0,077626}{2}\right)^4 \checkmark}{\frac{0,07626}{2}} = 12,264x \checkmark$$

$$5\,877\,003,611 = 12,264x \quad \mathbf{x = 479\,200,70 \checkmark\checkmark} \quad (18)$$

[26]

QUESTION 4

$$4.1 \quad \frac{1\,396 - 1\,300}{1\,300} \checkmark\checkmark = \mathbf{7,4\% \checkmark\checkmark} \quad (4)$$

$$4.2 \quad Q_{n+1} \checkmark = 1,05 \cdot \checkmark Q_n - 50, \checkmark \quad Q_0 = 6\,500 \checkmark \quad (4)$$

$$4.3 \quad \begin{array}{ll} A = 8\,020 & B = 8\,371 \checkmark \\ C = 8\,739 & D = 9\,126 \checkmark \\ E = 9\,532 & F = 9\,959 \checkmark \end{array} \quad (3)$$

$$4.4 \quad 9\,126 / 4 = 2\,281 < 2\,301 \checkmark \quad \mathbf{\text{during } 8^{\text{th}} \text{ year } \checkmark} \quad (2)$$

$$4.5 \quad \frac{2\,655 - 2\,472}{2\,472} \checkmark = 7,4\% \checkmark \quad \mathbf{\text{constant exponential growth; } \checkmark/\text{thus}} \quad (3)$$

Malthusian **[16]**

QUESTION 5

- 5.1 (a) prey $\approx 526\ 000$ ✓ predator $\approx 4\ 500$ ✓ (2)
- (b) $\pm 4\ 180 - 4190$ ✓✓ (2)
- (c) B ✓✓ (2)
- 5.2 $S_{n+1} = 4\ 000$ ✓ + 760 ✓ - $0,2$ ✓ $\times 4\ 000 = 3\ 960$ ✓✓ (5)
- 5.3 $2/3$ ✓ $\times 3 \times 8$ ✓ $\times 0,05$ ✓ = **0,8** ✓✓ (5)
- 5.4 $533\ 300$ ✓ = $500\ 000$ ✓ + $0,8$ ✓ $(500\ 000) \left(1 - \frac{500\ 000}{K}\right) - 0,4$ ✓ $(500\ 000)$
 $K = 1\ 200\ 000$ ✓✓ (6)
- [22]**

QUESTION 6

- 6.1 (a) 8^{th} ✓✓ (2)
- (b) A ✓✓ (2)
- 6.2 $\frac{2a+3b}{a+2b} = \frac{a+2b+a+b}{a+2b}$ ✓ = $1 + \frac{a+b}{a+2b}$ ✓ = $1 + \frac{1}{\frac{a+b+b}{a+b}}$ ✓ = $1 + \frac{1}{1 + \frac{b}{a+b}}$ ✓
- = $1 + \frac{1}{1 + \frac{1}{\frac{a+b}{b}}}$ ✓ = $1 + \frac{1}{1 + \frac{1}{1 + \frac{a}{b}}}$ ✓ = $1 + \frac{1}{1 + \frac{1}{1 + T_n}}$

OR

$$\begin{aligned} \frac{2a+4b-b}{a+2b} &= 2 - \frac{b}{a+2b} = 2 - \frac{1}{\frac{a+2b}{b}} \\ &= 2 - \frac{1}{2 + \frac{a}{b}} \\ &= 2 - \frac{1}{2 + T_n} \end{aligned} \quad (6)$$

[10]

Total for Module 3: 100 marks

MODULE 4 MATRICES AND GRAPH THEORY

QUESTION 1

1.1 (a) 1×1 ✓✓ (2)

(b) $(k+3 \quad k+2 \quad 3) \begin{pmatrix} k \\ 3 \\ 1 \end{pmatrix} = (k^2 + 3k) + (3k+6) + 3$

OR

$(k \quad 1 \quad 1) \begin{pmatrix} k+3 \\ 3k+2 \\ 7 \end{pmatrix} = (k^2 + 3k) + (3k+2) + 7$

$k^2 + 6k + 9 = 0 \quad \mathbf{k = -3}$ ✓✓ (8)

1.2 (a) 1 ✓✓

(b) **pr** ✓✓

(c) **27p** ✓✓

(d) **pr/q** ✓✓ (8)

[18]

QUESTION 2

2.1 $\begin{pmatrix} 1 & -3 & 6 & 4 \\ 0 & 3 & -1 & 1 \\ 0 & 2 & 2 & -18 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & -3 & 6 & 4 \\ 0 & 3 & -1 & 1 \\ 0 & 8 & 0 & -16 \end{pmatrix}$ ✓✓

$y = -2; \quad z = -7$ ✓

$x - 3(-2) + 6(-7) = 4 \quad \mathbf{x = 40}$ ✓ (6)

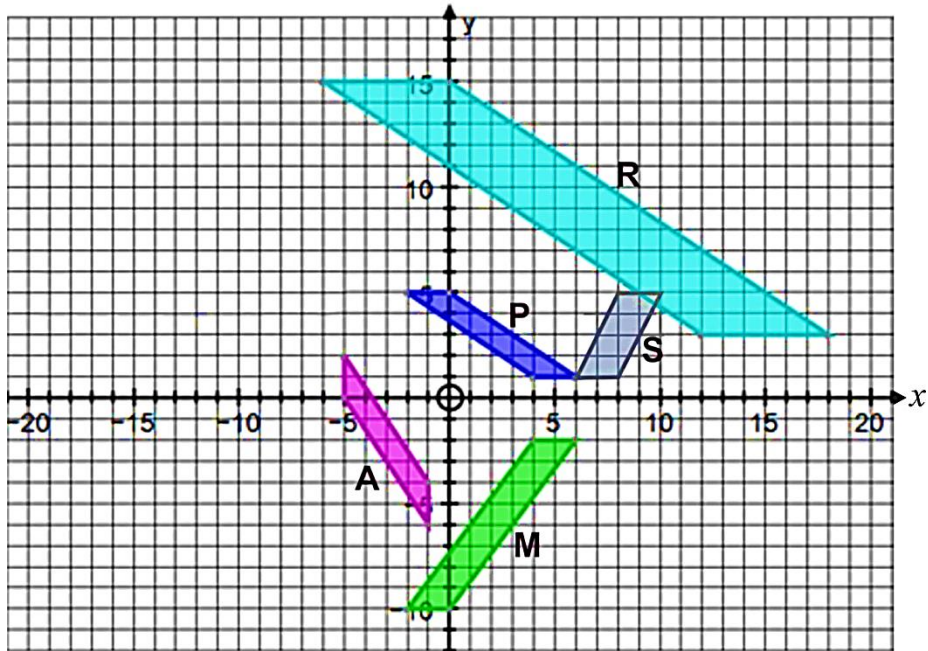
2.2 **L:** ✓✓ $0x + 0y + 0z \neq 1$ ✓ (equation is an inconsistency) (3)

2.3 **D:** ✓✓ $0x + 0y + 0z = 0$ ✓ (3)

[12]

QUESTION 3

3.1



- (a) Reflection; ✓ direction $y = -x$; ✓ coordinates ✓ (3)
- (b) Enlargement; ✓ scale $k = 3$; ✓ coordinates ✓ (3)
- (c) Stretch; ✓ invariant line $y = 0$; ✓ scale $k = 2$; ✓ coordinates ✓ (4)
- (d) Shear, ✓ invariant line $y = 0$; ✓ scale $k = 2$, ✓ coordinates ✓ (4)

3.2 $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 6 \\ 1 \end{pmatrix} = \begin{pmatrix} -5,55 + 10 \\ 5,15 - 1 \end{pmatrix} = \begin{pmatrix} 4,45 \\ 4,15 \end{pmatrix}$ ✓✓

$6\cos\theta - \sin\theta = 4,45$ ✓✓ AND $\cos\theta + 6\sin\theta = 4,15$ ✓✓

$\cos\theta = 0,83\ 378\ 378 \dots$ OR $\sin\theta = 0,55\ 270\ 270 \dots$

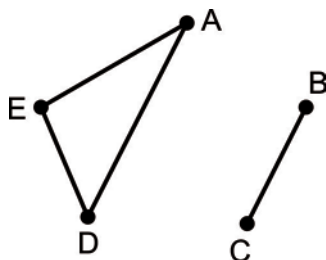
$\theta = 33,5^\circ$ $\theta = 33,6^\circ$ ✓✓ (12)

[26]

QUESTION 4

- 4.1 6 edges ✓✓ (2)
- 4.2 not symmetrical ✓✓ (2)
- 4.3 all vertices loop to themselves ✓✓ (2)

4.4



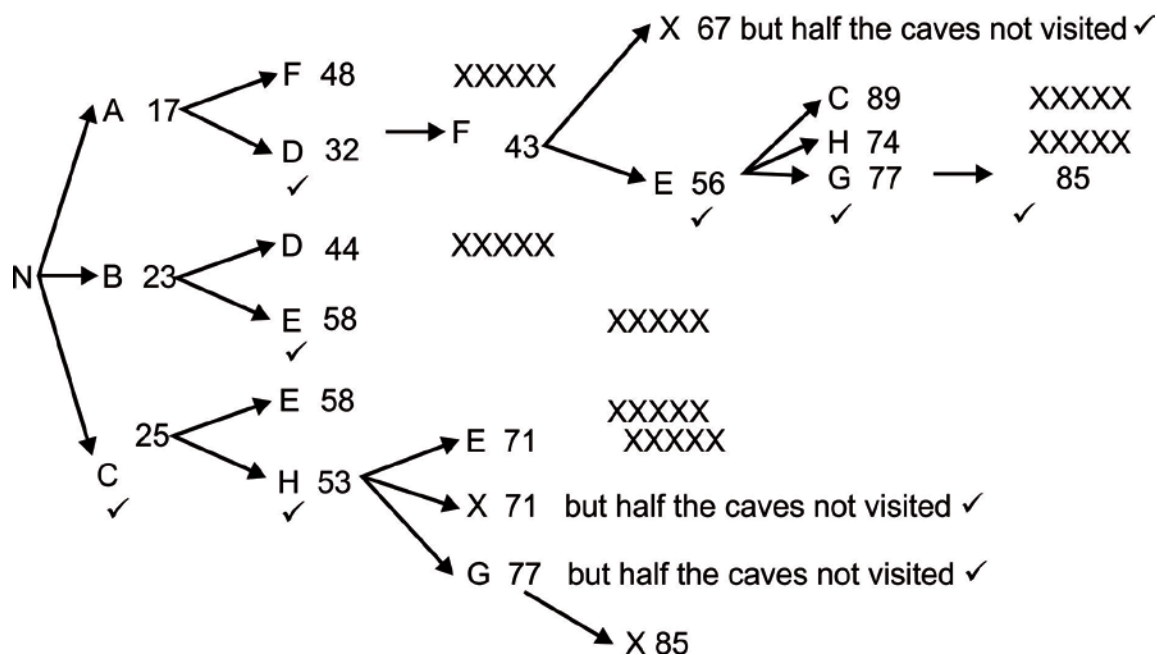
five vertices, four edges ✓✓
correct edges, disconnect ✓✓

(4)
[10]

QUESTION 5

- 5.1
- | | | |
|----|----|---|
| NA | 17 | |
| AD | 15 | |
| DF | 11 | ✓ |
| DB | 21 | ✓ |
| FX | 24 | ✓ |
| XG | 8 | ✓ |
| XH | 18 | ✓ |
| NC | 25 | ✓ |
| EF | 13 | ✓ |
| EH | 18 | ✓ |
- = 170 min ✓✓** (10)

5.2



$\therefore N A D F E G X \checkmark = 85 \text{ min } \checkmark$ (12)

OR

	A	B	C	D	E	F	G	H	X	
✓ N	N₁₇	N₂₃	N₂₅							
✓ A	N ₁₇	N ₂₃	N ₂₅	A₃₂	•	A₄₈				
✓ B	N ₁₇	N ₂₃	N ₂₅	B₄₄	B₅₈	A ₄₈				
✓ C	N ₁₇	N ₂₃	N ₂₅	A ₃₂	C₅₈	A ₄₈	•	C₅₃	•	
D	N ₁₇	N ₂₃	N ₂₅	A ₃₂	B/C ₅₈	D₄₃	•	C ₅₃	•	
✓ F	N ₁₇	N ₂₃	N ₂₅	A ₃₂	F₅₆	D ₄₃	•	C ₅₃	F₆₇	But half caves not visited ✓
H	N ₁₇	N ₂₃	N ₂₅	A ₃₂	H₇₁	D ₄₃	H₇₇	C ₅₃	H₇₁	But half caves not visited ✓
✓ E	N ₁₇	N ₂₃	N ₂₅	A ₃₂	F ₅₆	D ₄₃	E₇₇	C ₅₃	H ₇₁	
✓ G	N ₁₇	N ₂₃	N ₂₅	A ₃₂	F ₅₆	D ₄₃	H/E ₇₇	C ₅₃	G₈₅	
G	N ₁₇	N ₂₃	N ₂₅	A ₃₂	F ₅₆	D ₄₃	H ₇₇	C ₅₃	G ₈₅	But half caves not visited ✓
$\therefore N$	A	D	F	E	G	x	= 85 min ✓✓			

[22]

QUESTION 6

6.1 (a) $n - 1$ ✓✓ (2)

(b) $n(n - 1)$ ✓✓ (2)

6.2 (a) $ABDCA$ ✓✓ $ACDBA$ }
 $ACBDA$ ✓✓ $ADBCA$ } ✓ (5)

(b) $(n - 1)!$ ✓✓ (3)

[12]

Total for Module 4: 100 marks

Total: 100 marks