GRADE 12 EXAMINATION
NOVEMBER 2015

ADVANCED PROGRAMME MATHEMATICS
ELECTIVE MODULE: FINANCE AND MODELLING

Time: 1 hour | 100 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 5 pages and an Information Booklet of 3 pages (i – iii). Please check that your question paper is complete.
2. Non-programmable and non-graphical calculators may be used, unless otherwise indicated.
3. All necessary calculations must be clearly shown and writing should be legible.
4. Diagrams have not been drawn to scale.
5. Round off your answers to two decimal digits, unless otherwise indicated.

FINANCE AND MODELLING

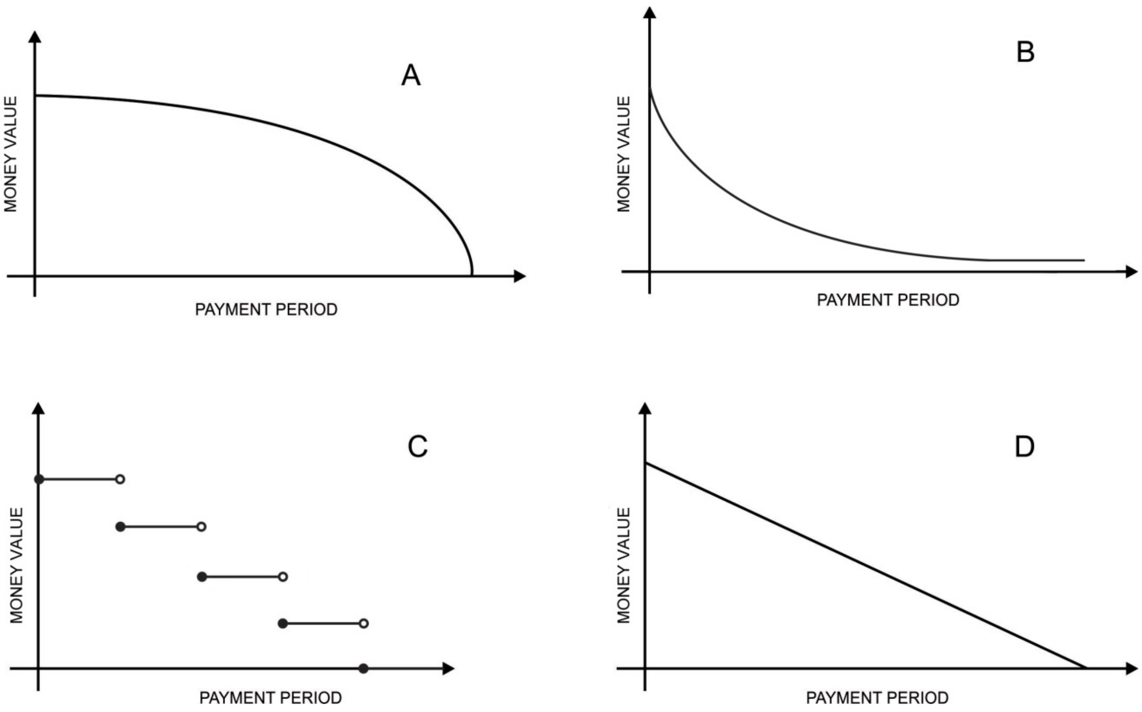
QUESTION 1

1.1 Four graphs are sketched below, each representing a different financial situation. Choose the graph that best represents the situation given in each statement below. Record only the letter of the graph.

(a) Outstanding balance on a home loan (2)

(b) Simple depreciation on office computers (2)

(c) Reducing balance on the depreciated value of a car (2)



1.2 The given recursive formula describes the annual depreciation of office equipment:

$$T_{n+1} = 0{,}86.T_n \quad \text{and} \quad T_0 = R780\ 000$$

(a) Give the initial value of the equipment. (1)

(b) Give the rate of depreciation as a percentage. (2)

(c) Give the value of the equipment in five years' time. (2)

[11]

QUESTION 2

Mandla obtains a loan from a bank for her new home. She is charged 7,2% interest per annum, compounded monthly. She intends repaying the loan (at the end of every month) with equal monthly instalments of R8 645 over a period of 15 years.

2.1 Calculate the value of her loan. (5)

2.2 Calculate how much of her first instalment is used to pay off loan capital. (4)

2.3 Calculate the equivalent annual interest rate, compounded daily, as a percentage, correct to four decimal places. (8)

[17]

QUESTION 3

The bank has approved a R1 200 000 home loan for Wikus. He is charged interest at 6,24% per annum, compounded monthly. He intends to pay off the loan in equal monthly instalments (at the end of each month) over a period of 20 years.

3.1 The bank insists that his monthly repayments may not exceed one-third of his monthly salary. Calculate the minimum **annual** income he needs to earn. (8)

3.2 Without being required to increase his monthly instalment, Wikus withdrew R17 000 just after his 115th payment. Calculate the balance outstanding on his loan immediately after his 128th payment. Assume his monthly repayment to be R8 764. (8)

3.3 Wikus missed his 129th and 130th payments. This forced him to increase his instalments to R9 200 per month from the 131st payment. Assuming that the outstanding balance after the 128th payment is R761 000, calculate the value of the final payment (which is less than R9 200), which will amortise his home loan. (10)

[26]

QUESTION 4

4.1 In 1915, the world's population was estimated to be 1,3 billion people. By 1950, it had almost doubled to 2,5 billion and by 1985, it had again doubled to just over 5 billion.

(a) Explain from these figures why it could be assumed that a Malthusian model adequately described the world's population growth. (2)

(b) Assuming a Malthusian model, the annual population growth rate is 2%, with the average lifespan of a person being 56 years. Calculate (as a percentage, correct to two decimal places) the annual birth rate. (4)

4.2 A more realistic model to predict the world's population is a Logistic model. The table below records the world's population at 35-year intervals, according to a particular Logistic model:

Year	**Population in Billions (P)**	**ΔP**	**ΔP/P**
1915	1,30		
1950	2,52	0,054 571	A
1985	5,12	B	C
2020	7,22	D	0,009 893
2055	10,12		

(a) Complete the table by calculating the values A, B, C and D, correct to six decimal places. (8)

(b) Calculate (as a percentage, correct to two decimal places) the annual intrinsic growth rate for this model. (4)

[18]

QUESTION 5

In the world-renowned Kruger National Park live both lion (L_n) and wildebeest (W_n). The latter, despite their size, are rather docile and readily fall prey to lions. The Lotka-Volterra model for these two species is given below:

$$W_{n+1} = W_n + 0{,}345.\, W_n \left(1 - \frac{W_n}{25\ 000}\right) - 0{,}000\ 655.\, W_n . L_n \quad \text{and}$$
$$L_{n+1} = L_n + 0{,}000\ 000\ 169.\, W_n . L_n - 0{,}083\ 333 . L_n$$

5.1 Give the average lifespan of a lion, in years. (2)

5.2 Calculate, correct to six decimal places, the efficacy rate at which lions turn their food into offspring. (4)

5.3 A female wildebeest gives birth once a year to one calf. The survival rate of a calf is about 60%. Calculate, correct to one decimal place, what percentage of the wildebeest is female. (4)

5.4 The population of these two species tends to an equilibrium. Calculate the equilibrium population for lion, if the equilibrium population for wildebeest is 5 500. (8)

[18]

QUESTION 6

6.1 Calculate the values of the next four terms generated by the recursive formula:

$$T_{n+1} = \frac{1}{2}\left[T_n + \sqrt{5.(T_n)^2 + 4.(-1)^n}\right] \quad \text{and} \quad T_1 = 1$$ (6)

6.2 A recursive equation is said to be **stable** if its final value is unaffected by the initial value.
A second order recursive equation $T_n = p.T_{n-1} + q.T_{n-2}$ is stable if it satisfies two conditions: $q < 1$ and $|p| < q + 1$.

Determine through calculations if the recursive equation $T_n = -2.T_{n-1} + 0{,}8.T_{n-2}$ is stable, or not. (4)

[10]

Total: 100 marks