GRADE 12 EXAMINATION
NOVEMBER 2014

ADVANCED PROGRAMME MATHEMATICS

MARKING GUIDELINES

Time: 3 hours 300 marks

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MODULE 1 CALCULUS AND ALGEBRA

QUESTION 1

Step 1: Let $n = 1$

$LHS = 1^2 = RHS$

Step 2: Assume true for $n = k$

then $1 + 3 + 5 + ... + (2k - 1) = k^2$

Step 3: For $n = k + 1$

$1 + 3 + 5 + ... + (2k - 1) + (2k + 1)$

$= k^2 + 2k + 1$

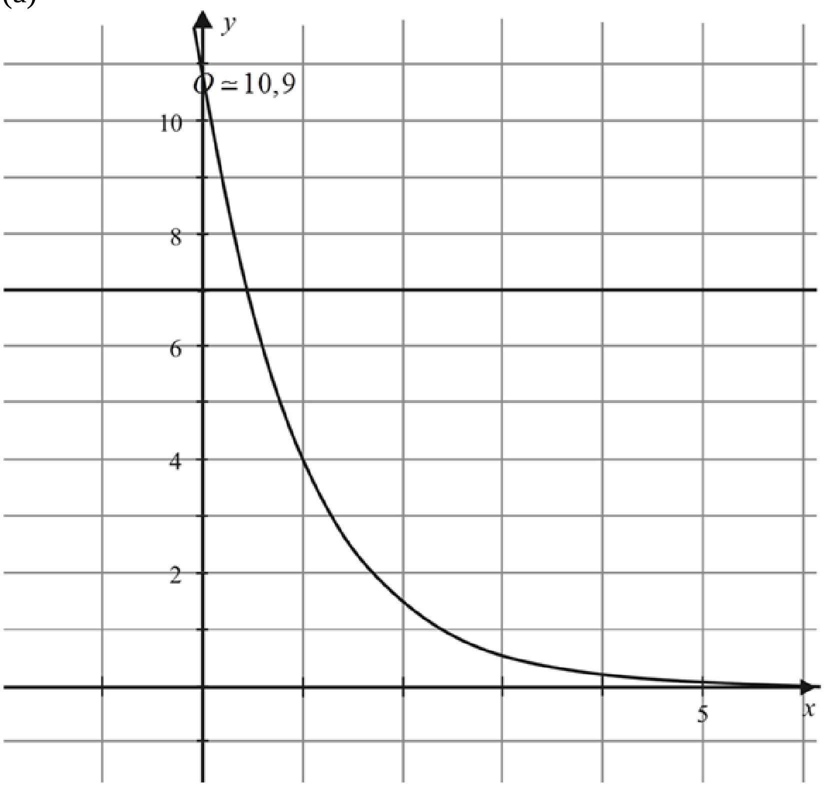
$= (k + 1)^2$

$\therefore$ by the P.M.I. the statement is true for $n \in \mathbb{N}$

[12]

QUESTION 2

2.1 (a)



(y-intercept, shape, asymptote, (1;4)

(8)

(b) $4e^{1-x} > 7$

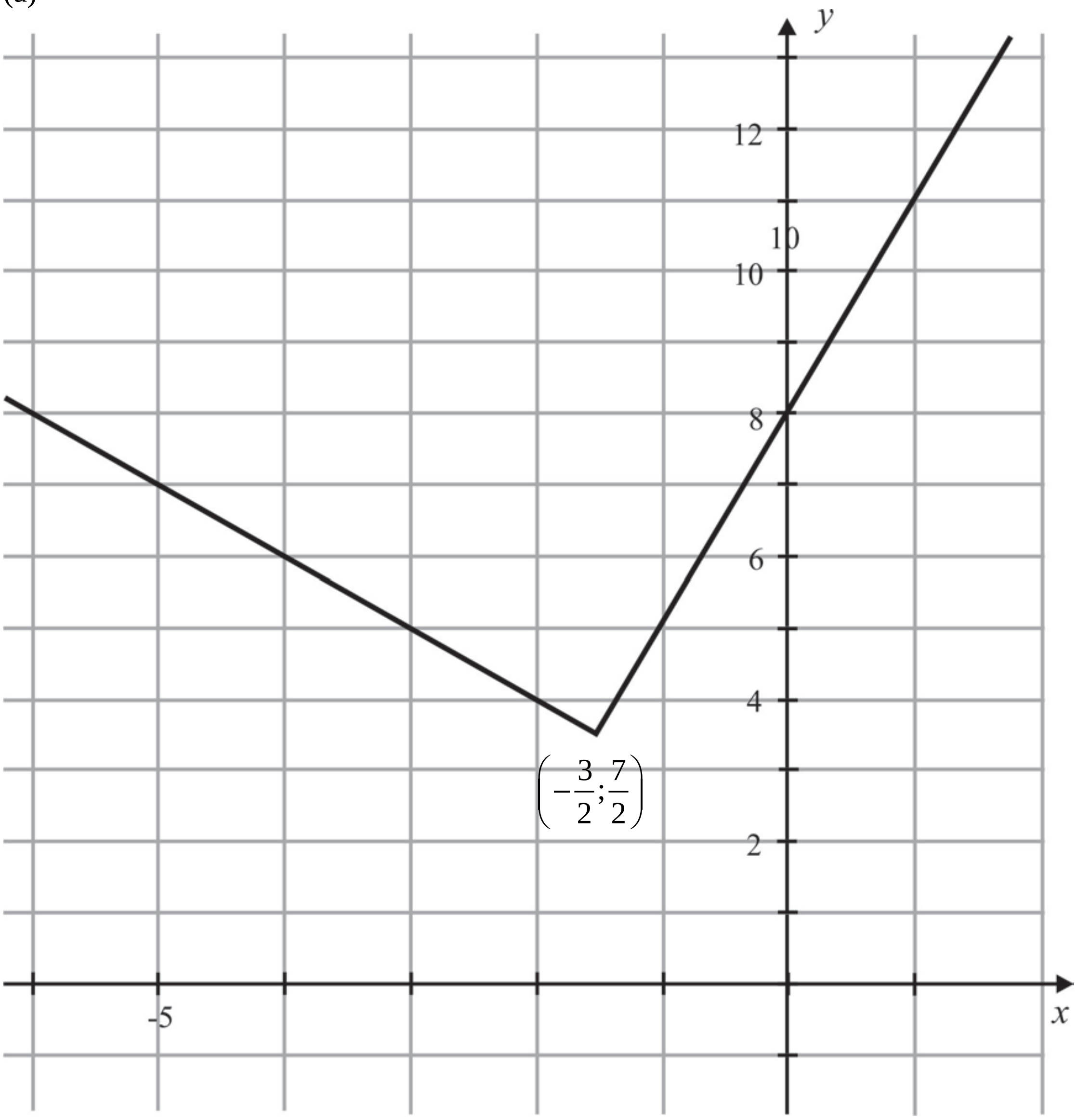
$e^{1-x} > 1.75$

$ln1.75 < 1 - x$

$0.56 < 1 - x$

$x < 0.44$ (6)

2.2 (a)



(8)

(b) For $g(x) \neq f(x)$ require $-1 \leq k \leq 3$ (4)
(note change of inequality)

[26]

QUESTION 3

3.1 $2(5-2i)-i(6i-1)$

$=10-4i-6i^2+i$ (note change of mark allocations)

$=16 \quad -3i$ (6)

3.2 $\dfrac{x-10}{(2x-1)(x+3)}=\dfrac{A}{2x-1}+\dfrac{B}{x+3}=\dfrac{Ax+3A+2Bx-B}{(2x-1)(x+3)}$ (note change of mark allocations)

$\therefore\ x: A+2B=1$

Const: +3A – B = –10

$\therefore\ \text{A}=-\dfrac{19}{7} \quad \text{B}=\dfrac{13}{7} \quad \text{thus } \dfrac{x-10}{(2x-1)(x+3)}=\dfrac{-\frac{19}{7}}{2x-1}+\dfrac{\frac{13}{7}}{x-3}$ (10)

3.3 $\lim_{n\to\infty}\dfrac{4\Sigma k-3\Sigma 1}{n^2}=\lim_{n\to\infty}\dfrac{1}{n^2}\left(\dfrac{4n(n+1)}{2}-3n\right)$

$=\lim_{n\to\infty}\left(2-\dfrac{1}{n}\right)=2$ (8)

[24]

QUESTION 4

4.1 Since $f\,'(x)>0\ \therefore\ f$ is always increasing

(a) Greatest value of $f(x)$ is at x = F (4)

(b) $f''(x)=0\ \therefore$ points of inflection at (and $f(x)\neq 0$) (note change of mark allocations)

x = B; C; E (6)

4.2 No. We have no indication of what the y-values are for the graph. (No given points) (4)

4.3 $f\,'(x)\neq 0$ thus $f(x)$ has no turning points. (4)

[18]

QUESTION 5

5.1 $y = (\sin 2x)(2-x)^{-3}$ (note change of mark allocations)

$\therefore \frac{dy}{dx} = 2\cos 2x(2-x)^{-3} + 3(2-x)^{-4}.\sin 2x$

OR $y = \frac{\sin 2x}{(2-x)^3}$

$\therefore \frac{dy}{dx} = \frac{2\cos 2x(2-x)^3 + 3(2-x)^2.\sin 2x}{(2-x)^6}$ (6)

5.2 (a) $1^3 + 2^3 - 1\times 2^2 = 1+8-4 = 5$

$\therefore$ point $(1\ ;\ 2)$ lies on curve. (2)

(b) $3x^2 + 3y^2\frac{dy}{dx} - \left(1.y^2 + 2y\frac{dy}{dx}.x\right) = 0$

$3x^2 + 3y^2\frac{dy}{dx} - y^2 - 2xy\frac{dy}{dx} = 0$

$\therefore \frac{dy}{dx}\left(3y^2 - 2xy\right) = y^2 - 3x^2$

$\therefore \frac{dy}{dx} = \frac{y^2 - 3x^2}{3y^2 - 2xy}$ (10)

(c) at (1 ; 2) $\frac{dy}{dx} = \frac{4-3}{12-4} = \frac{1}{8}$

$\therefore y = \frac{1}{8}x + c$

$2 = \frac{1}{8} + c$

$c = 1\frac{7}{8}$

$\therefore$ tangent $y = \frac{1}{8}x + 1\frac{7}{8}$ (4)

5.3 (a) $y = (\mathrm{b}x + \mathrm{c})^{-1}$

$$\therefore \frac{dy}{dx} = -1(bx+c)^{-2}.b = -b(bx+c)^{-2}$$

$$\frac{d^2y}{dx^2} = 2b(bx+c)^{-3}.b = 2b^2(bx+c)^{-3}$$

$$\frac{d^3y}{dx^3} = -6b^2(bx+c)^{-4}.b = -6b^3(bx+c)^{-4}$$ (6)

(b) $\frac{d^ny}{dx^n} = (-1)^n n!b^n (bx+c)^{-(n+1)}$ (6)

[34]

QUESTION 6

6.1 $SA = 2rh + r\alpha h + 2\times\frac{1}{2}r^2\alpha$ but $r = h$ $\therefore A = 2r^2 + r^2\alpha + r^2\alpha$ (note change of mark allocations)

$= 2r^2(\alpha+1)$ (8)

6.2 $Vol = \frac{1}{2}r^2\alpha h$

If $r = h$ and total volume 1000 cm^3

Then $1000 = \frac{1}{2}r^3\alpha$ (note change of mark allocations)

Thus $\alpha = \frac{2000}{r^3}$ (4)

6.3 $\therefore A = 2r^2\left(1+\frac{2000}{r^3}\right) = 2r^2 + \frac{4000}{r}$

$\therefore \frac{dSA}{dr} = 4r - \frac{4000}{r^2} = 0$ for minimum SA

$\therefore 4r^3 - 4000 = 0$

$\therefore r = 10$ and thus $\alpha = 2$ (8)

[20]

QUESTION 7

7.1 (a) Vertical asymptotes: $(x+5)(x+1)=0$

$\therefore x=-5 \quad or \quad x=-1$ (4)

(b) $2x^3+6x^2-25x-39=\left(x^2+6x+5\right)(2x-6)+x-9$

$\therefore \dfrac{2x^3+6x^2-25x-39}{x^2+6x+5}=2x-6+\dfrac{x-9}{x^2+6x+5}$ (this step not necessary)

$\therefore$ oblique asymptote $=y=2x-6$ (8)

(note change in mark allocation)

(c) $f''(x)>0=$ Implies concave up

$\therefore -5<x<-1$ (4)

7.2 (a) $f(x)=\cos x-0,25x$ initial estimate

$f'(x)=-\sin x-0,25$

$\therefore x_{n+1}=x_n+\dfrac{\cos x-0,25x}{\sin x+0,25}$

$x=-2,1333$ (8)

(b) If $f'(x)=0$ i.e. $\sin x=-0,25$

$or \quad x=\sin^{-1}\left(-\dfrac{1}{4}\right)$

Algebraically ÷ by 0

OR Graphically will not cut x-axis (2)

[26]

QUESTION 8

8.1 Let $u=3x$ $\therefore \dfrac{du}{3}=dx$

$\therefore \dfrac{1}{3}\int \cos^2 u\,du \quad =\dfrac{1}{3}\int \dfrac{1}{2}(1+\cos 2u)\,du$

$=\dfrac{1}{6}\left(u+\dfrac{\sin 2u}{2}\right)+c$

$=\dfrac{x}{2}+\dfrac{\sin 6x}{12}+c$

(8)

8.2 $\int \cos 2\theta \sin 5\theta d\theta$

$$= \frac{1}{2}\int \sin(5\theta + 2\theta) + \sin(5\theta - 2\theta) d\theta$$

$$= \frac{1}{2}\int \sin 7\theta + \sin 3\theta d\theta$$

$$= \frac{1}{2}\left[\frac{-\cos 7\theta}{7} - \frac{\cos 3\theta}{3}\right] + c$$

$$= \frac{-\cos 7\theta}{14} - \frac{\cos 3\theta}{6} + c$$ (6)

8.3 Let $u = 2 - x$ then $x = 2 - u$

$du = -dx$

$$\therefore -\int \frac{10 - 5u}{u^{\frac{1}{2}}} du \qquad = -\int 10u^{-\frac{1}{2}} - 5u^{\frac{1}{2}} du$$

$$= \frac{-10u^{\frac{1}{2}}}{\frac{1}{2}} + \frac{5u^{\frac{3}{2}}}{\frac{3}{2}} + c$$

(note changes of sign in last 3 steps)

$$= -20u^{\frac{1}{2}} + \frac{10}{3}u^{\frac{3}{2}} + c$$

$$= -20(2 - x)^{\frac{1}{2}} + \frac{10}{3}(2 - x)^{\frac{3}{2}} + c$$ (8)

[22]

QUESTION 9

9.1 $\therefore \text{V} = \pi\int_0^4 \left[\frac{(x-4)^2}{4}\right]^2 dx = \frac{\pi}{16}\int_0^4 (x-4)^4 dx = \frac{\pi}{16}\left|\frac{(x-4)^5}{5}\right|_0^4 = \frac{\pi}{16} \times \frac{1024}{5} = \frac{64}{5}\pi \text{ units}^3$ (8)

9.2 No The line $y = x$, not an axis of symmetry $\therefore$ rotation would be different. Or any other valid reason. (4)

9.3 $\pi\int_0^a \frac{(x+4)^4}{16} dx = \frac{\pi}{16}\left[\frac{(x-4)^5}{5}\right]_0^a = 10\pi$

$$= \frac{\pi}{80}\left[(a-4)^5 - \left(-4)^5\right)\right] = 10\pi$$

$$\therefore (a-4)^5 = 800 - 1024$$

$$\therefore a - 4 = -2{,}9515$$

$$\therefore a = 1{,}0485$$ (6)

[18]

Total for Module 1: 200 marks

MODULE 2 STATISTICS

QUESTION 1

1.1 (a) $X \sim N\left(38,4\ ;\ 4,6^2\right)$

$$P\left(30 < x < 40\right) = P\left(\frac{30-38,4}{4,6} < z < \frac{40-38,4}{4,6}\right)$$
$$= P\left(-1,83 < z < 0,35\right)$$
$$= 0,4664 + 0,1368$$
$$= 0,6032$$

(8)

(b) $P\left(x > k\right) = 0,9$

$z = -1,28$

$$-1,28 = \frac{k-38,4}{4.6}$$

$k = 32,512$

(8)

1.2 (a) $2,33\left(\sqrt{\frac{(0,32)(0,68)}{n}}\right) \leq 0,1$

$$\sqrt{\frac{0,2176}{n}} \leq 0,0429$$

$n \geq 119$

(6)

(b) A 98% CI for p is

$$0,32 \pm 2,33\left(\sqrt{\frac{(0,32)(0,68)}{119}}\right)$$

$(0,2204\ ;\ 0,4196)$

(4)

[26]

QUESTION 2

2.1 (a) False
(b) True
(c) False
(d) True
(e) True
(f) False

(6)

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2.2 $H_0 : \mu = 8$
$H_1 : \mu < 8$

Rejection Region:
Reject H_0 if $z < -1,75$

Test Statistic:

$$z = \frac{7,92 - 8}{\frac{0,2}{\sqrt{30}}} = -2,19$$

Conclusion:
Since $z < -1,75$ we reject H_0 at the 4% level of significance and suggest sufficient evidence to support the claim. (10)

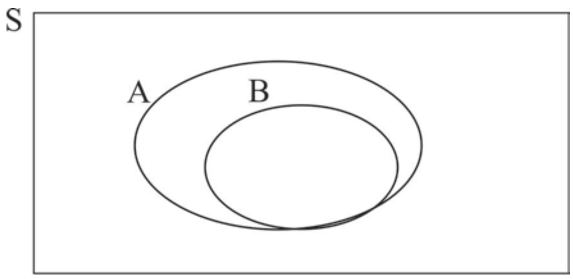
[16]

QUESTION 3

3.1 (a) A and B are independent. (2)

(b) $P(A \cap B) = 0 \therefore$ A and B are mutually exclusive. (2)

3.2



$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$1 = \frac{P(A \cap B)}{P(B)}$$

$$P(B) = P(A \cap B)$$

(4)

3.3 (a) $\dfrac{\binom{5}{4}\binom{4}{0}}{\binom{9}{4}} = \dfrac{5}{126} = 0,0400$ or $\dfrac{5}{9} \times \dfrac{4}{8} \times \dfrac{3}{7} \times \dfrac{2}{6} = \dfrac{5}{126} = 0,0400$ (8)

(b) $\binom{5}{4} + \binom{5}{3} + \binom{5}{2} + \binom{5}{1} = 30$ (8)

3.4 $P(x > 4) = \binom{6}{5}(0,6)^5(0,4)^1 + \binom{6}{6}(0,6)^6(0,4)^0 = 0,2333$ (8)

[32]

QUESTION 4

4.1 (a)

x	1^1	2	3
$P(x)$	$\frac{7}{24}$	$\frac{8}{24}$	$\frac{9}{24}$

$\frac{7}{24}+\frac{8}{24}+\frac{9}{24}=1$

$\therefore P(X=x)$ is a probability mass function. (4)

(b) $\bar{x}=1\left(\frac{7}{24}\right)+2\left(\frac{8}{24}\right)+3\left(\frac{9}{24}\right)$

$=\frac{25}{12}$ (5)

4.2 (a) $P(correct)=1(0,65)+\frac{1}{5}(0,35)$

$= 0,72$ (4)

(b) $P(guesses/correct)=\dfrac{\frac{1}{5}(0,35)}{0,65+\frac{1}{5}(0,35)}$

$=\frac{7}{72}=0,0972$ (4)

[17]

QUESTION 5

5.1 (a) Sub $(21;\bar{y})$ into $y=-7x+163$

$\bar{y}=-7(21)+163$

$=16$ (2)

(b) $y=-7(5)+163$

$= 128$ (2)

5.2 $r=-\frac{1}{7}=-0,1429$

A correlation coefficient is negative and considered very weak. (3)

5.3 The estimate is unreliable as the correlation is too weak. (2)

[9]

Total for Module 2: 100 marks

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MODULE 3 FINANCES AND MODELLING

QUESTION 1

1.1 $T_{n+1} = T_n \bullet \left(1 + \frac{0,09}{4}\right) - 11\ 300,\ T_0 = 150\ 000$ (5)

1.2 $0,5 = (1-c)^4 \qquad \therefore c = 15,91\%$

$0,1 = (1-0,1591)^n \quad \therefore n = 13,3 \quad \therefore n = 14$ years (9)

[14]

QUESTION 2

2.1 $OB = \dfrac{8932,75\left[1-\left(1+\frac{0,0648}{12}\right)^{-51}\right]}{\frac{0,0648}{12}} = \mathbf{397\ 290{,}48}$ (6)

2.2 $400\ 000\left(1+\frac{0,0674}{12}\right)^3 = \dfrac{x\left[1-\left(1+\frac{0,0674}{12}\right)^{-48}\right]}{\frac{0,0674}{12}}$

$\boldsymbol{x = 9\ 691{,}81}$ (8)

2.3 $8\ 932,75 \times 189 + 9\ 691,81 \times 48 - 1\ 200\ 000 = \mathbf{953\ \ 496{,}63}$ (6)

[20]

QUESTION 3

3.1 $\left(1+\frac{0,058}{4}\right)^4 = \left(1+\frac{i}{12}\right)^{12}$

$1 + \frac{i}{12} = 1,004\ 810\ ...$

***i* = 5,7722%** per annum, compound monthly (8)

3.2 $850000 = \dfrac{7\ 300\left[1-\left(1+\frac{0,057722}{12}\right)^{-n}\right]}{\frac{0,057722}{12}}$

$0,43991 = 1,00481^{-n}$

$n = 171,128$

180 – 172 = **8 months** earlier (10)

[18]

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QUESTION 4

4.1 0,95 = 1 + 0,6 – x **x = 0,65**
500 = – 2 500 + y **y = 3 000** (6)

4.2 $F_{n+1} = 0{,}95.F_n + 500$, $F_0 = 5\ 000$ **F_{20} = 8 207 or 8 208** (rounding)
Population increasing , but at slower rate as time goes on. (5)

4.3 $F_n = 0{,}95^{100}.5000 + \dfrac{500[0{,}95^{100} - 1]}{0{,}95 - 1}$ (k) (n) (c, K) **= 9 970** (5)

[16]

QUESTION 5

5.1 (a) prey: accept 18 000 - 20 000 predator: accept 700 - 1 000 (2)

(b) 1 000 < prey < 94 000 (accept 100 – 2 000; 93 000 – 94 000) (2)

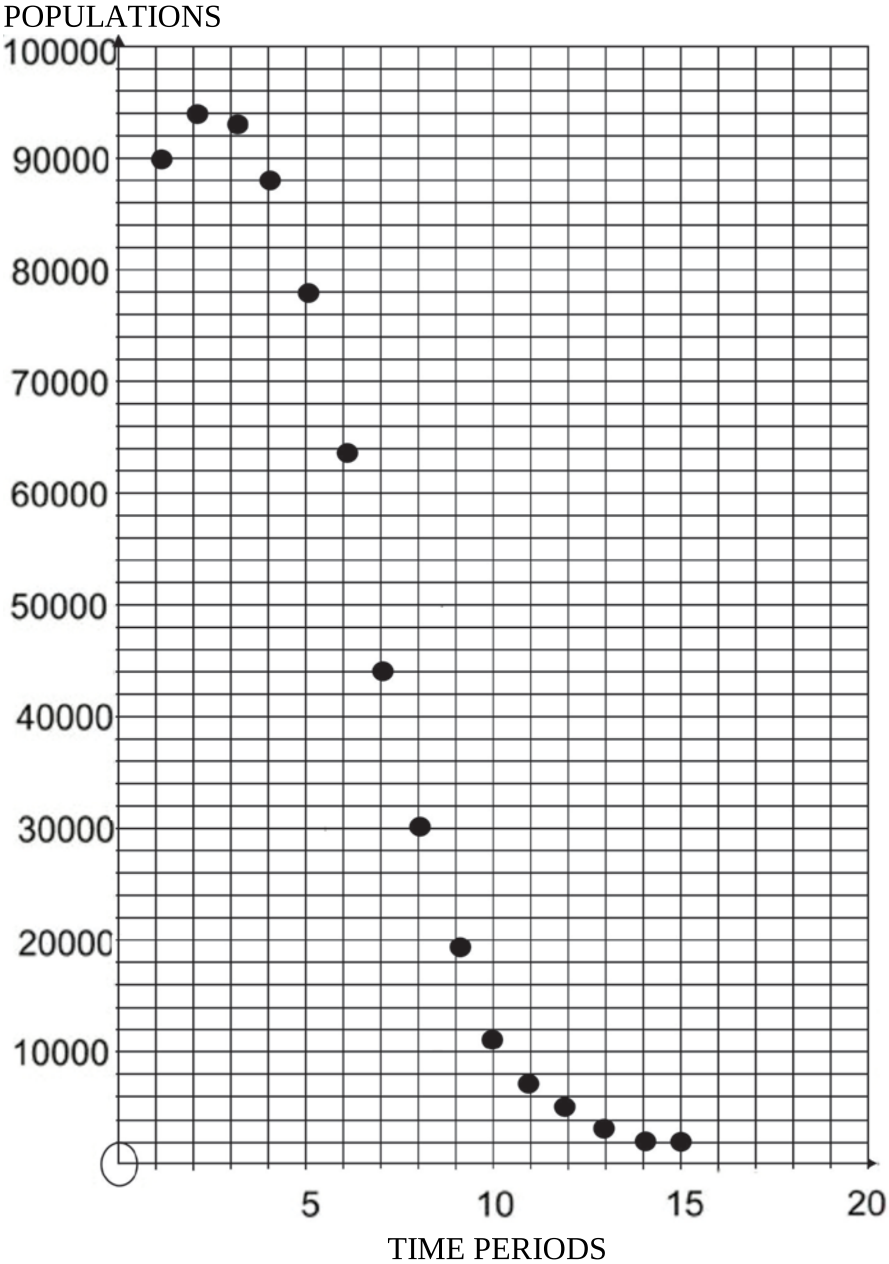
(c) quadrant 3 (2)

(d) rate of deadly interactions between predator and prey (2)

(e) Could lead to extinction of prey
at some time points in cycles prey population is already very low (1 000)
and now has a higher rate of fatalities per cycle

OR
prey decreases more rapidly per cycle
double amount of kills per cycle and hence potentially more predators

OR
Equilibrium point of prey half the original
R = c/(bf) so denom doubled, hence value halved (4)



3 initial points discrete plot accuracy of plots (8)

[20]

QUESTION 6

6.1 $\boldsymbol{Q_{n+1} = 0{,}12P_n + 0{,}9Q_n + 400\ 000}$ (4)

6.2 For equilibrium, $P_{n+1} = P_n$: $-0{,}2P + 0{,}06Q = -600\ 000.$

For equilibrium, $Q_{n+1} = Q_n$: $0{,}12P - 0{,}1Q = -400\ 000.$

$\boldsymbol{P = 6\ 562\ 500, \quad Q = 11\ 875\ 000}$ (8)

[12]

Total for Module 3: 100 marks

MODULE 4: MATRICES AND GRAPH THEORY

QUESTION 1

1.1 (a) $4a - (-2) = 0$ **a = – ½** (4)

(b) $\frac{1}{6} \begin{pmatrix} 4 & 1 \\ 2 & 2 \end{pmatrix}$ (4)

(c) $A - 2B = \begin{pmatrix} a & 2 \\ -1 & 4 \end{pmatrix} - \begin{pmatrix} 4 & -2 \\ -4 & 2b \end{pmatrix}$

$a - 4 = -7$ **a = – 3**

$4 - 2b = 4a = -12$ **b = 8** (6)

1.2 (a) q x p (2)

(b) p x r (2)

(c) r x r (2)

[20]

QUESTION 2

2.1 (a) $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ (2)

(b) $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}$ (shear) (factor) (direction) $= \begin{pmatrix} -3 & -1 \\ -1 & 0 \end{pmatrix}$ (6)

2.2 $360^{o} \div 8 = 45^{o}$

$\begin{pmatrix} cos\,45 & -sin\,45 \\ \sin 45 & \cos 45 \end{pmatrix} \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} -2{,}121 \\ 0{,}707 \end{pmatrix}$ (6)

2.3 $\begin{pmatrix} \cos 2A & \sin 2A \\ \sin 2A & -\cos 2A \end{pmatrix} = \begin{pmatrix} -0{,}8 & -0{,}6 \\ -0{,}6 & 0{,}8 \end{pmatrix}$

$\cos 2A = -0{,}8$ AND $\sin 2A = -0{,}6$

$2A = 180° + 36{,}87° = 216{,}87$ so $A = 108{,}435°$

$\tan A = -3$ **y = –3x** (10)

[24]

QUESTION 3

3.1 det = 16 (2)

3.1 $\begin{pmatrix} 4 & 2 & 6 \\ -1 & 2 & 2 \\ 0 & 1 & 3 \end{pmatrix}$ (2)

3.2 $a = 2\times2-2\times6=-8$

$b = 4\times3-0\times6=12$

$c = 4\times2-(-1)\times2=10$ (6)

[10]

QUESTION 4

4.1 Dijkstra (2)

4.2 Prim (2)
4.3 Fleury (2)

4.4 Nearest Neighbour (2)

[8]

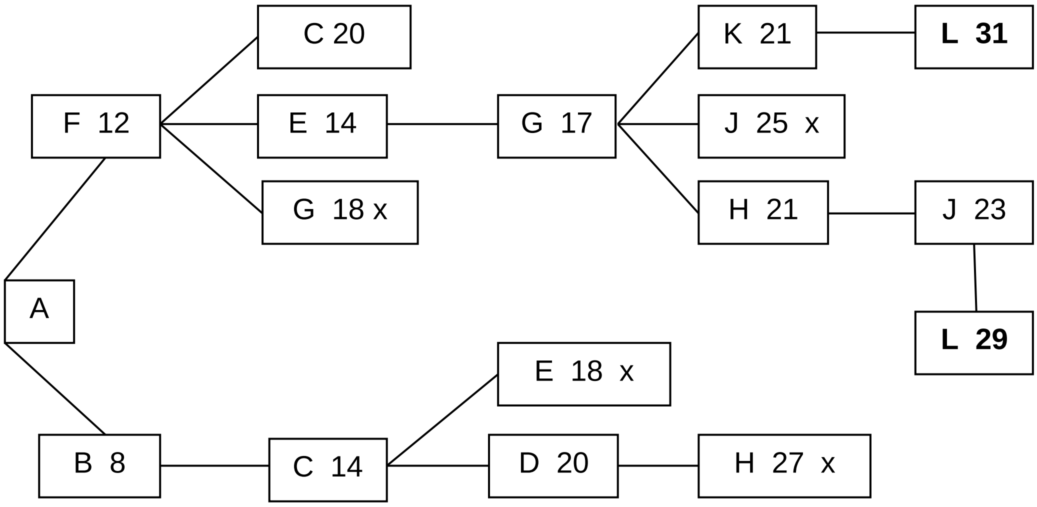
QUESTION 5

5.1

	A	B	C	D	E	F	G	H	J	K	L
A											
B	8										
C		6									
D			6								
E			4								
F	12		8		2						
G					3	6					
H				7			4				
J							8	2			
K							4				
L									6	10	

(6)

5.2



A F E G H J L = 29

OR

A		B,C,D,E,F,G,H,J,K,L
AB	(8)	C,D,E,F,G,H,J,K,L
ABF	(12)	C,D,E,G,H,J,K,L
ABFC or ABFCE	(14)	D,G,H,J,K,L
ABFCEG	(17)	D,H,J,K,L
ABFCEGD	(20)	H,J,K,L
ABFCEGDH or ABFCEGDHK	(21)	J,L
ABFCEGDHKJ	(23)	L
ABFCEGDHKJL	(29)	

A F E G H J L = 29 (12)

5.3 EG HJ (4)

5.4 (a) 11 vertices, therefore each vertex must be connected to 5 other vertices.
True only of G. (2)

(b) converse not true
OR
Theorem gives one case of when there will be HC; it is not exhaustive (2)

[26]

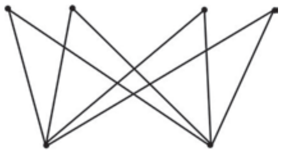
QUESTION 6

6.1 (a) m + n (2)

(b) m.n (2)

6.2 1 x 10 = 11 or 2 x 5 = 7 (4)

6.3



vertices
edges (4)

[12]

Total for Module 4: 100 marks

Total: 300 marks

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