

ADVANCED PROGRAMME MATHEMATICS: PAPER II

Time: 1 hour

100 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 15 pages, and an Information Booklet of 4 pages (i–iv). Please check that your question paper is complete.

2. This question paper consists of THREE modules:

Choose **ONE** of the **THREE** modules:

MODULE 2: STATISTICS (100 marks) OR

MODULE 3: FINANCE AND MODELLING (100 marks) OR

MODULE 4: MATRICES AND GRAPH THEORY (100 marks)

3. Non-programmable and non-graphical calculators may be used.

4. All necessary calculations must be clearly shown and writing should be legible.

5. Diagrams have not been drawn to scale.

6. **Rounding of final answers.**

MODULE 2: Four decimal places, unless otherwise stated.

MODULE 3: Two decimal places, unless otherwise stated.

MODULE 4: Two decimal places, unless otherwise stated.

MODULE 2 STATISTICS

QUESTION 1

1.1 It is known that 5,7% of girls in South Africa between the ages of 12 and 17 are diagnosed with ADD (attention deficit disorder). There are 28 students in a class in a high school for girls. 12 students are randomly selected from the class. What is the probability that:

(a) No more than 1 of these 12 students are diagnosed with ADD? (7)

(b) No one in the class is diagnosed with ADD, given that none of the 12 students selected, were diagnosed with ADD? (4)

1.2 A bag contains blue balls and red balls. Balls are drawn randomly from the bag. The probability that at least 3 of the balls drawn will be blue is given by:

$$1 - \sum_{k=0}^a \frac{\binom{12}{k} \binom{8}{f(k)}}{\binom{x}{7}}$$

(a) Is drawing being done with or without replacement? Motivate. (2)

(b) How many balls are in the bag? (2)

(c) How many balls are being drawn from the bag? (1)

(d) How many of the balls in the bag are red? (2)

(e) Write down the value of *a*. (2)

(f) Write down **two different** possible expressions for *f(k)*. (2)

[22]

QUESTION 2

2.1 In 1999 (the year when most of you were born), "Statistics South Africa" tells us that a rounded-off figure of 1 400 000 babies were born in South Africa. The weight of these babies were normally distributed with a mean of 3,2 kg and a standard deviation of 0,85 kg.

- (a) How many babies would you estimate to weigh above 2,8 kg? (7)
- (b) Low birth weight (LBW) is the weight recorded by babies who are born below the 10th percentile. Determine the weight below which a baby would be considered a LBW baby. (7)

2.2 A survey is conducted to determine the mean mass of 64 women between the age of 30 and 40. The results produce a confidence interval of (59 kg; 63 kg) with a standard deviation of 9 kg.

- (a) What is the sample mean? (2)
- (b) Determine the level of confidence, to the nearest percentage, that the interval contains the true mean. (8)

[24]

QUESTION 3

The Omar Fuel Company claims that the additive added to their fuel reduces the average fuel consumption. The Smart company claims that the additive has no effect on the average fuel consumption. To defend their claim the Omar Fuel Company gathered the following information:

	FUEL WITH ADDITIVE	FUEL WITHOUT ADDITIVE
MEAN	7,2 litres/100 km	8,1 litres/100 km
STANDARD DEVIATION	2,85	2
SAMPLE SIZE	35	38

Use a hypothesis test to evaluate the claim made by the Omar Fuel Company at a 5% level of significance.

[9]

QUESTION 4

The heights (in cm) of males between the ages of 8 and 18 are recorded. Each male is represented as an ordered pair (x, y) , where x is the age of the male and y his height. The following results are obtained:

$$\sum x = 161 \quad \sum y = 1910 \quad \sum xy = 26270 \quad \sum x^2 = 2293 \quad \bar{y} = 159\frac{1}{6}$$

The correlation coefficient was found to be 0,9389.

- 4.1 Show how you can conclude that 12 male heights were recorded. (2)
- 4.2 Determine the equation of the regression line. (6)
- 4.3 Comment on the strength of the correlation between the heights and ages recorded. (2)
- 4.4 The regression equation predicts a height of 210 cm for a 24 year old male. Is this result reliable? Explain. (2)
- [12]**

QUESTION 5

The probability density function for the time taken (in minutes) by students to complete a test is given by:

$$f(x) = \begin{cases} a(x-30)^2 & 30 \leq x \leq 60 \\ 0 & \text{elsewhere} \end{cases}$$

- 5.1 Show that $a = \frac{1}{9\,000}$. (7)
- 5.2 What is the median time taken by students to complete the test? Round to the nearest minute. (7)
- [14]**

QUESTION 6

6.1 Two events, A and B, are such that:

- A and B are independent.
- $P(A) = x$
- $P(B) = y$

With the aid of a Venn diagram, prove that A and B' are also independent. (8)

6.2 A medical committee of 5 is chosen from 6 doctors, 3 dentists and 7 others. Determine the number of ways the committee can be selected if it must contain at least one member who is either a doctor or a dentist. (4)

6.3 Jamielee is moving to a new house and she has bought a triangular display cabinet with three shelves so that she can display her crystal animals given to her by her grandmother. She has 11 animals. The top shelf of the display cabinet can hold 2, the middle shelf 4 and the bottom shelf 6 ornaments. In how many ways can she display these animals? (note: they are displayed randomly on each shelf) (7)

[19]

Total for Module 2: 100 marks

MODULE 3 FINANCE AND MODELLING

QUESTION 1

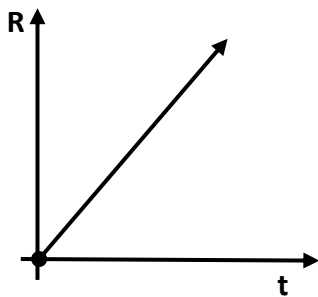
1.1 In South Africa, VAT is calculated at 15% on taxable goods and services.

(a) Olive recently paid R5 640 to have her car serviced; this amount included VAT. Calculate what the service itself cost, that is, without VAT. (2)

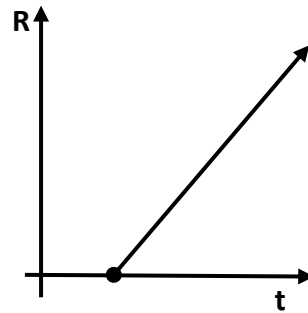
(b) VAT was recently increased from 14%. Calculate the percentage, correct to two decimal places, by which the cost of goods and services rose. (3)

1.2 The value of an investment trebles every 24 months. Interest is compounded monthly. Calculate, to the nearest month, the time over which the investment doubles. (5)

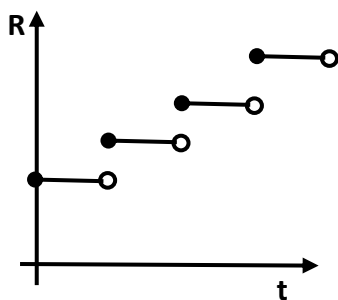
1.3 Four graphs are given below, with Rands (R) sketched against time (t). Match each of these graphs to one of the five financial descriptions given. Each description may be used a maximum of once.



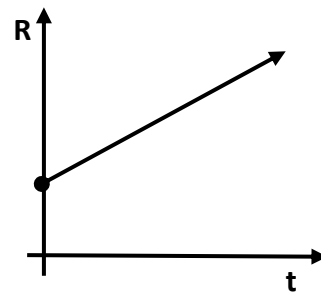
Graph A



Graph B



Graph C



Graph D

1. The cost of parking at a shopping mall, charged in intervals of time.
2. The profit of a new restaurant, given that initially income defrays start-up costs and consequently profit only starts showing in a few months' time.
3. The total simple interest earned over time on a once-off deposit.
4. The total compound interest earned over time on a once-off deposit.
5. A plumber's bill is directly proportional to the time he spends at a customer's house, plus an initial call-out fee that is charged.

(8)
[18]

QUESTION 2

Harold takes out a loan of R500 000 at an annual interest rate of 8,8%, compounded monthly.

- 2.1 Harold plans to amortise the loan by paying R2 500 per month, starting in one month's time. Explain, with the aid of calculations, why the bank would not accept him repaying R2 500 per month. (3)
- 2.2 The bank instructs him to pay off the loan over a period of 8 years with equal monthly instalments, starting one month after the loan is approved. Calculate the value of these equal monthly payments. (5)
- 2.3 Harold decides instead to pay off the loan with 95 monthly payments of R7 300. Calculate the value of the final 96th payment, which will be less than R7 300. (12)
- [20]**

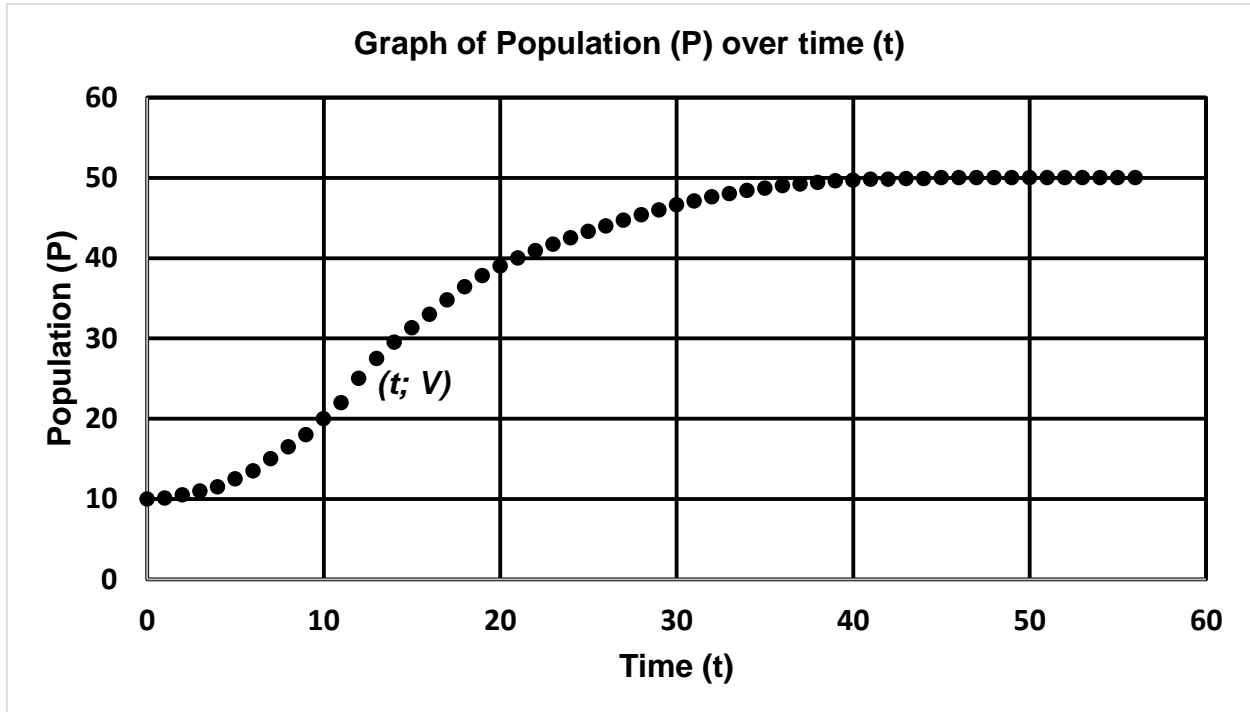
QUESTION 3

An amount of x rands is invested in an account which earns interest at 8% per annum, compounded monthly. After six years, one-third of the accumulated money is withdrawn and deposited into a separate account that earns interest at 10% per annum, compounded quarterly. The remainder is left in the original account, still earning interest at 8% per annum, compounded monthly. At the end of a further two years, the combined value of the two accounts is R20 702,50. Calculate (to the nearest rand) the value of x , the original deposit.

[12]

QUESTION 4

A discrete population model is sketched below, with Population (y -axis) and Time (x -axis). The point $(t; V)$ is the point on the curve at which the concavity of the curve changes.



- 4.1 State, with a reason, the type of population model this graph represents. (2)
 - 4.2 Write down the value V . (2)
 - 4.3 The model has a line of best fit for $\frac{\Delta P}{P}$ (y -axis) against P (x -axis) with a regression equation $\frac{\Delta P}{P} = -0,0025P + r$. Calculate the value of r , the intrinsic growth rate of the population, correct to three decimal places. (4)
 - 4.4 Using $r = 0,13$ determine the value of t . (6)
- [14]**

QUESTION 5

In the Karoo regions of South Africa, the humble dassie easily falls prey to the mighty eagle. Their **annual** populations are governed by the Lotka-Volterra models given below:

$$D_{n+1} = D_n + a.D_n \left(1 - \frac{D_n}{18\,000} \right) - 6\,000, \quad D_0 = 12\,000 \quad \text{and}$$

$$E_{n+1} = E_n + f.b.D_n.E_n - 0,1.E_n, \quad E_0 = 30$$

5.1 Consider the term $f.b.D_n.E_n$, that appears in the predator formula.

- (a) Explain what this term represents. (2)
- (b) Explain what the parameter f represents. (2)
- (c) Calculate, correct to four decimal places, the value of f , if $f.b.D_n.E_n = 15$. (3)

5.2 Half the population of dassies is female, and each female has a litter every eight months. Each litter has three kits and the survival rate of the kits is 67%. Calculate the value of the **annual** intrinsic growth rate of the dassies. (5)

5.3 These populations tend to an equilibrium. Calculate the equilibrium population of the dassies. Assume from Question 5.1 (c) that $f = 0,003$. (10)

[22]

QUESTION 6

The Sierpinski triangle is a well-known fractal design, based on a series of equilateral triangles.

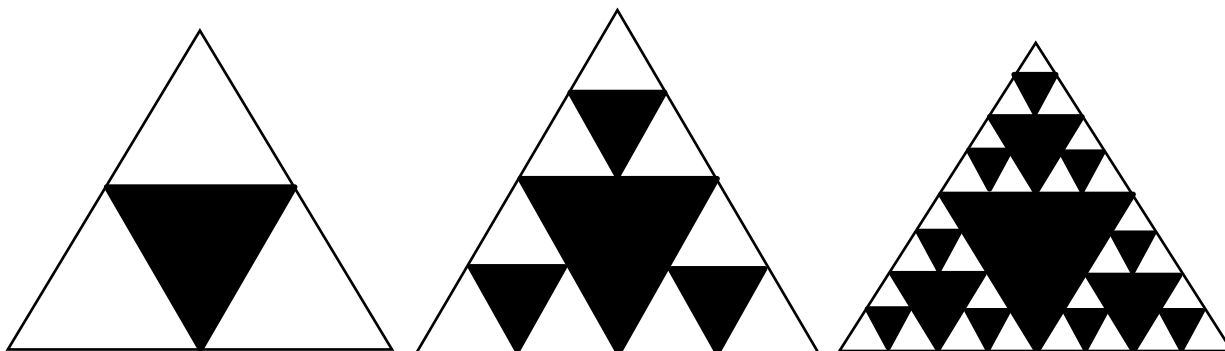


FIGURE 1

FIGURE 2

FIGURE 3

In FIGURE 1, the midpoints of each side of the triangle have been joined to form a smaller, inner triangle, which has been shaded.

In FIGURE 2, the midpoints of each of the sides of the unshaded triangles have again been joined to create smaller, inner triangles, which have also been shaded.

In FIGURE 3, the process of joining midpoints and shading triangles is repeated.

6.1 The **areas** of the shaded triangles in each successive figure are $16\sqrt{3}$, $28\sqrt{3}$ and $37\sqrt{3}$. The formula $T_{n+1} = \frac{7 \cdot T_n}{4} - \frac{3 \cdot T_{n-1}}{4}$ generates the **areas** of the shaded triangles.

(a) Find the area (correct to three decimal places) of the shaded triangles in the sixth figure. (4)

(b) Determine the sum to infinity (correct to one decimal place) of the area of the shaded triangles. (2)

6.2 The (internal and external) **perimeters** of the **shaded triangles** in the first four figures are 24, 60, 114 and 195 units respectively. Find a second-order recursive formula that generates the perimeter of all the shaded triangles in FIGURE n. (8)

[14]

Total for Module 3: 100 marks

MODULE 4 MATRICES AND GRAPH THEORY**QUESTION 1**

1.1 Consider the matrices $P = \begin{pmatrix} 3 & 6 & 2 & -2 \\ 0 & -1 & 4 & 6 \end{pmatrix}$ and $Q = \begin{pmatrix} 3 & 6 \\ -2 & -1 \\ 0 & 5 \\ -7 & 0 \end{pmatrix}$

Determine the matrix product PQ . (4)

1.2 Use Gaussian Reduction to solve x , y and z in the following set of simultaneous equations. Do **not** merely write down the answers; be sure to indicate row reductions.

$$3x + 2y = 11 \qquad x - 2z = 0 \qquad 6y + 4z = 5 \qquad (8)$$

1.3 The **trace** of a matrix is the sum of the elements of the leading diagonal. Give the trace of:

(a) the 3×3 identity matrix. (2)

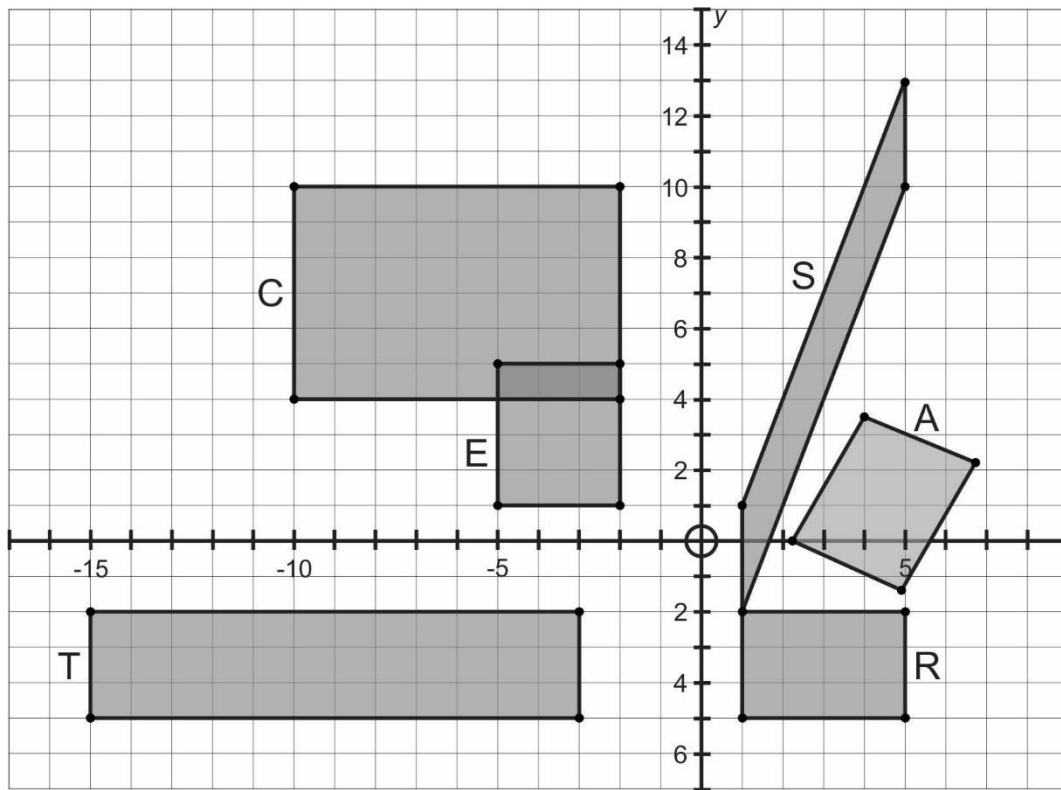
(b) the transformation matrix for a reflection. (2)

(c) the transpose of a matrix, whose trace is t . (2)

[18]

QUESTION 2

In the sketch below, six figures (A, C, E, S, T and R) are given. All are transformations of each other.



- 2.1 Given E is the image of R: describe the transformation in words. (2)
 - 2.2 Given S is the image of R: state the factor by which R was sheared, parallel to the y-axis. (2)
 - 2.3 Given C is the image of S: state the factor by which the area of S has been enlarged. (4)
 - 2.4 Given T is the image of C: calculate a single transformation matrix that will map C directly onto T. (6)
 - 2.5 Given A is the image of R, after an anticlockwise rotation: calculate (correct to two decimal places) the acute angle of rotation, if the image of vertex (5; -2) now lies at (4,025; 3,578). (10)
- [24]**

QUESTION 3

The inverse of the 3×3 matrix $\begin{pmatrix} 1 & 2 & -4 \\ 6 & 2 & 1 \\ 5 & 1 & 0 \end{pmatrix}$ is to be found using diagonalisation.

The first three steps have been given correctly:

STEP 1: $\begin{pmatrix} 1 & 2 & -4 & 1 & 0 & 0 \\ 6 & 2 & 1 & 0 & 1 & 0 \\ 5 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$ Row 1
Row 2
Row 3

STEP 2: $\begin{pmatrix} 1 & 2 & -4 & 1 & 0 & 0 \\ 0 & -10 & 25 & -6 & 1 & 0 \\ 0 & -9 & 20 & -5 & 0 & 1 \end{pmatrix}$ Row 1
Row 2 – 6.Row 1
Row 3 – 5.Row 1

STEP 3: $\begin{pmatrix} 5 & 0 & 5 & -1 & 1 & 0 \\ 0 & -10 & 25 & -6 & 1 & 0 \\ 0 & 0 & -25 & 4 & -9 & 10 \end{pmatrix}$ 5.Row 1 + Row 2
Row 2
10.Row 3 – 9.Row 2

- 3.1 Write down the determinant of the original matrix. (2)
 - 3.2 Complete the process of diagonalisation. (6)
 - 3.3 Hence write down the inverse of the matrix, where the elements of the matrix are given as integer values. (4)
- [12]**

QUESTION 4

Consider a simple, connected graph with 8 vertices. Four vertices have degree 6, two have degree 4, one has degree 1 and the degree of the eighth vertex is unknown.

- 4.1 Briefly explain why this graph will not contain an Eulerian circuit. (2)
 - 4.2 Will this graph contain an Eulerian path? Give a reason for your answer. (2)
 - 4.3 If the graph contains a Hamiltonian circuit, how many edges will this circuit contain? (2)
 - 4.4 Determine the degree of the eighth vertex, if the graph has 19 edges. (4)
- [10]**

QUESTION 5

The given graph consists of 8 vertices and 14 edges, with the relevant weight indicated on each edge.

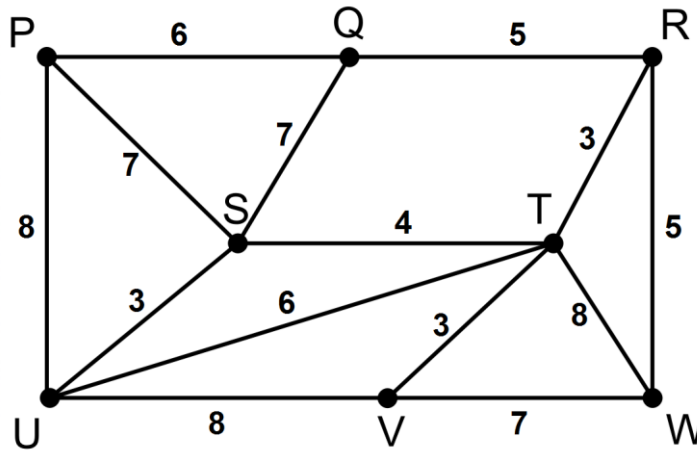


Table 1 below represents all the Lower Bounds from vertex R. The vertex mentioned is the vertex initially left out when calculating the Lower Bound from R.

TABLE 1: LOWER BOUNDS FROM VERTEX R

P	Q	S	T	U	V	W
36	36	35	37	35	36	36

Table 2 below represents an incomplete list of Upper Bounds from the various vertices.

TABLE 2: UPPER BOUNDS FROM EACH VERTEX

P	Q	R	S	T	U	V	W
47	47	unknown	42	42	41	48	45

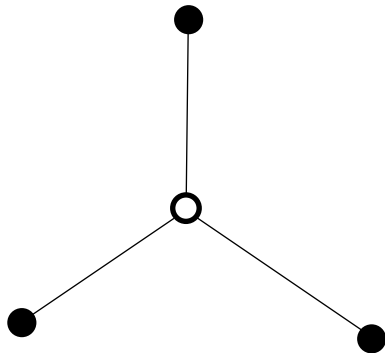
- 5.1 Design a spanning tree of minimum weight using Prim’s Algorithm, starting at vertex T. Be sure to indicate the order in which you choose the edges. Also clearly indicate the total length of this spanning tree. (8)
- 5.2 Complete Table 2 by calculating the Upper Bound from vertex R. Use the Nearest Neighbour Algorithm, and clearly show the order in which you choose the edges. (10)
- 5.3 If x represents the weight of a "good route", refer to the tables above and explain why $37 \leq x \leq 41$. (2)
- 5.4 By inspection, find a "good route" for the graph, starting at vertex R. Carefully record the edges you choose, as well as the weight of the route. (8)

[28]

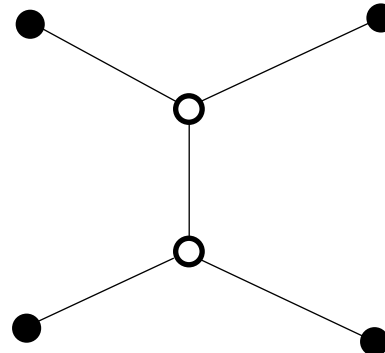
QUESTION 6

The **Steiner Tree Problem** is named after the Swiss mathematician Jakob Steiner (1796–1863).

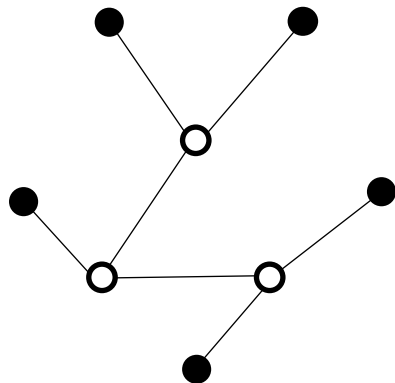
For any graph consisting of n vertices, the vertices are to only be **indirectly** connected to each other by using inserted Steiner vertices. The result is always a tree. Four specific cases are given below. The solid dots represent the original vertices of the graph, and the open rings represent the inserted Steiner vertices.



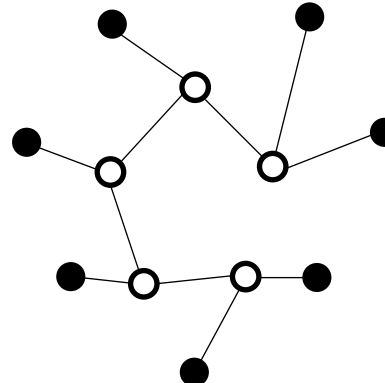
3 original vertices and
1 Steiner vertex



4 original vertices and
2 Steiner vertices



5 original vertices and
3 Steiner vertices



7 original vertices and
5 Steiner vertices

- 6.1 Give the degree of each Steiner vertex. (2)
 - 6.2 State the relationship between the number of original vertices n , and the number of edges e . (2)
 - 6.3 Sketch a possible Steiner tree in a graph with 6 original vertices. Use solid dots for the original vertices, and open rings for the Steiner vertices. (4)
- [8]**

Total for Module 4: 100 marks